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WELLBORE BEHAVIOR DURING SATURATED STEAM INJECTION

BY

PETER H. HOLST

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES
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FACULTY OF ENGINEERING

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EDMONTON, ALBERTA

NOVEMBER, 1965

CHAPTER IV

THEORY OF THE EARTH AND ITS HISTORY

1840

1841

CHAPTER V

THEORY OF THE EARTH AND ITS HISTORY
CONTINUED

1842

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CHAPTER VI

THEORY OF THE EARTH AND ITS HISTORY
CONTINUED

1844

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance a thesis entitled WELLBORE BEHAVIOR DURING SATURATED STEAM INJECTION submitted by Peter H. Holst in partial fulfilment of the requirements for the degree of Master of Science in Chemical Engineering.

ABSTRACT

A mathematical model was formulated to describe the injection of saturated steam down oilwell tubing under constant inlet conditions.

The system was divided into three parts, consisting of the fluid, the wellbore and the formation. Each part was considered to be a separate system, with the heat flux across the boundaries acting as the coupling parameter. The wellbore consisted of a tubing string enclosed by either one or two casing strings. The heat transfer in the wellbore was considered steady state. The heat transfer in the formation was treated as unsteady state radial conduction and the fluid flow was described by a modified two phase flow correlation.

The analysis resulted in three equations, two of them implicit, which were subsequently solved simultaneously on the 7040 computer. The solution did not present any convergence problems. Although a complete verification of the mathematical model was not possible, the calculated temperature profiles were similar in shape to an observed temperature profile.



ACKNOWLEDGEMENT

The author wishes to express his sincere appreciation for the guidance and encouragement of Dr. D.L. Flock, Professor, Department of Chemical and Petroleum Engineering, University of Alberta, under whose supervision this investigation was performed.

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INTRODUCTION

With the cost of finding and developing high grade crude reserves continually increasing, more attention is being focused on the development of high viscosity or "heavy" oil reserves. Since the oil viscosity is highly dependent on temperature, a small increase in reservoir temperature decreases the oil viscosity markedly, facilitating increased production rates and ultimate recoveries. One of the more popular means of increasing the reservoir temperature is steam injection. To evaluate the feasibility of a thermal project, a reasonably accurate estimate of the heat losses from the wellbore, sandface pressure and sandface quality of the steam would be desirable, if not necessary. Since the injection times are generally quite small, steady state solutions would not be very representative and unsteady state solutions are required.

The wellbore heat transfer coefficient contains radiation and natural convection terms, both of which are temperature dependent. The nature of the radiation and natural convection contributions present major difficulties in deriving an unsteady state equation describing the heat transfer in the wellbore, and it appears that a numerical solution is the best that could be hoped for. To avoid lengthy, time consuming numerical techniques, the problem was divided into parts; whereby, wellbore heat transfer was con-

THEORY

The first part of the theory is the basic principles of the theory. The second part is the application of the theory to the practice. The third part is the conclusion of the theory. The fourth part is the summary of the theory. The fifth part is the appendix of the theory. The sixth part is the bibliography of the theory. The seventh part is the index of the theory. The eighth part is the list of figures of the theory. The ninth part is the list of tables of the theory. The tenth part is the list of references of the theory. The eleventh part is the list of abbreviations of the theory. The twelfth part is the list of symbols of the theory. The thirteenth part is the list of units of the theory. The fourteenth part is the list of constants of the theory. The fifteenth part is the list of variables of the theory. The sixteenth part is the list of parameters of the theory. The seventeenth part is the list of functions of the theory. 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sidered steady state and formation heat transfer was treated as unsteady state radial conduction. The fluid had to be considered since flowing friction affects the fluid temperature, which in turn affects the heat losses. The three parts were then solved simultaneously, with the heat flux across the various boundaries serving as a common boundary condition. (Heat lost by the fluid = heat transferred through the well-bore = heat entering the formation.)

The heat losses result in an enthalpy change in the fluid causing a temperature drop in a single phase fluid or a quality change in a two phase steam. Other investigators (11, 13, 15) have treated the single phase case. Satter(13) considered the two phase case neglecting friction and kinetic energy effects, thus obtaining a constant steam temperature in the tubing. Under certain conditions this is a poor assumption, as was substantiated with field data(3) (see Figure 4), where a temperature drop of 44°F was recorded for a 1600 ft. well.

This investigation was limited to an attempt to treat the injection of a saturated steam at constant inlet conditions (quality, flowrate, temperature) as fully as possible. The resulting generalized computer program is subject to the assumptions and approximations, which are believed to be realistic, made in obtaining the mathematical model. The accuracy of the results is largely dependent on the accuracy of the empirical correlations employed in the solution.

LITERATURE REVIEW

Two authors who have concerned themselves with the specific problem of heat losses from the wellbore during hot fluid injection are Ramey(11) and Satter(13). Since their work formed the basis of this investigation, it is treated thoroughly in this review.

Ramey was primarily concerned with the temperature profile during single phase fluid injection. Application of his model to saturated steam leads to a simple relationship subject to considerable error. Ramey's basic assumption may be listed as follows:

- 1) the physical properties of the fluid and the formation are independent of depth and temperature.
- 2) the heat transfer in the wellbore is rapid compared to the rate of heat transfer in the earth, and may be considered steady state.
- 3) the overall heat transfer coefficient is independent of depth, and may be determined prior to a temperature calculation.
- 4) the frictional losses and the kinetic energy effects are negligible.

With the application of energy balances, simplified by the stated assumptions, Ramey(11) developed equations describing the temperature of the injected fluid as a function of depth. For the liquid he obtained:

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$$T_s = a + bz - bA + (T_{s_o} + bA - a) e^{-z/A} \quad (1)$$

and for the gas obtained:

$$T_s = a + bz - A \left(b + \frac{1}{JC_p} \right) + \left[T_{s_o} - a + A \left(b + \frac{1}{JC_p} \right) \right] e^{-z/A} \quad (2)$$

where

$$T_E = a + bz \quad (3)$$

$$A = \frac{W_{st} C_p (k_E + r_c U_o f(t))}{2\pi r_c U_o k_E} \quad (4)$$

The time function $f(t)$ is a function introduced to make equation 9 valid. Ramey presented a graph of $f(t)$ versus dimensionless time (t_D), with U_T as a parameter. (Refer to Appendix I-C for the derivation of $f(t)$)

The equations were developed applying an energy balance on an element of fluid of length dz at a depth of z ft. in the tubing, and equating the heat lost by the fluid to the heat transferred through the wellbore and the heat entering the formation.

Thus

$$\Delta Q_1 = \Delta Q_2 = \Delta Q_3 \quad (5)$$

† Refer to the "Nomenclature" for the definition of terms.

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where

ΔQ_1 = the heat lost by the fluid (Btu/hr)

ΔQ_2 = the heat transferred through the wellbore (Btu/hr)

ΔQ_3 = the heat entering the formation (Btu/hr)

For an incompressible liquid flowing through a constant diameter tubing, the energy balance reduces to:

$$W_{st} \left(dH - \frac{g}{g_c} \frac{dz}{J} \right) = -\Delta Q_1 \quad (6)$$

Assuming that $C_v \approx C_p$ and that the friction losses are negligible, application of thermodynamics yields:

$$\Delta Q_1 = -W_{st} C_p dT_s \quad (7)$$

Also have

$$\Delta Q_2 = 2\pi r_c U_o (T_s - T_c) dz \quad (8)$$

and

$$\Delta Q_3 = \frac{2\pi k_E (T_c - T_E)}{f(t)} \quad (9)$$

Equating ΔQ_2 and ΔQ_3 , solving for T_c , substituting the expression for T_c into equation 8 and subsequently equating ΔQ_1 and ΔQ_2 yields:

$$\frac{\partial T_s}{\partial z} + \frac{T_s}{A} - \frac{T_E}{A} = 0 \quad (10)$$

where A is defined by equation 4 and $\neq 0$. The solution of equation 10, subject to the boundary condition $T_s(0, t) = T_{s_o}$,

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results in equation 1.

For an ideal gas, $dH = C_p dT_s$, thus:

$$\Delta Q_1 = -W_{st} (C_p dT_s + \frac{g}{g_c} \frac{dz}{J}) \quad (11)$$

Substituting equation 11 for equation 7, and following the same procedure as outlined for the liquid case, results in the derivation of equation 2.

The rate of heat loss per foot of wellbore may be expressed by:

$$q_r = \frac{\Delta Q_2}{dz} = \frac{2\pi r_c k_E (T_s - T_E)}{r_c U_o f(t) + k_E} \quad (12)$$

If T_s and U_o are known, as would be the case for the injection of a saturated steam with negligible friction losses, the calculation of q_r at any depth is straight forward.

Satter(13) improved Ramey's(11) solution by circumventing the restrictions of a constant wellbore heat transfer coefficient and temperature independent fluid properties. By limiting the integration of equation 9 to small increments in depth, Satter could choose increments over which the wellbore heat transfer coefficient and fluid properties are relatively constant. The overall wellbore heat transfer coefficient was represented by:

$$\frac{1}{U_o} = \frac{1}{h_s} + \frac{r_2 - r_1}{k_l} + \frac{1}{h_c + h_r} \quad (13)$$

where

$$\begin{aligned}h_c &= \text{convection coefficient for the annulus} \\&= F_a (T_s, T_c) \\h_r &= \text{radiation coefficient for the annulus} \\&= F_b (T_s, T_c)\end{aligned}$$

For single phase flow, U_o is clearly dependent on the fluid temperature, while the fluid temperature is dependent on U_o (see equation 1 and 2). A trial and error solution is required to obtain a value for the fluid temperature and the wellbore heat transfer coefficient simultaneously. Satter solved for the temperature profile by straight iteration, whereby the first trial temperature is assumed, then a temperature is calculated. If the assumed and calculated temperatures do not agree to within a specified error, the calculated value becomes the new trial value.

For saturated steam Satter obtained relationships for the quality versus depth and percent heat loss versus depth.

From equation 6 obtain

$$-\frac{\Delta Q_1}{W_{st}} = dH - \frac{g}{g_c} \frac{dz}{J} \quad (14)$$

For a saturated steam at a constant temperature

$$dH = H_v \cdot dy \quad (15)$$

The substitution of equation 15 and equation 12 into equation 14 yields:

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$$\frac{\partial y}{\partial z} - B' + \frac{T_s - (a + bz)}{A'} = 0 \quad (16)$$

where

$$A' = \frac{W_{st} H_v (r_c U_o f(t) + k_E)}{2\pi r_c k_E U_o} \quad (17)$$

$$B' = \frac{g}{g_c H_v J} \quad (18)$$

$$T_E = a + bz$$

The integration of equation 16 from $z - \Delta z$ to z , yields the equation for the steam quality.

$$\begin{aligned} y(z,t) = y(z - \Delta z, t) &+ \frac{b\Delta z^2}{2A'} \\ &+ \frac{[A'B' + a + b(z - \Delta z) - T_s]\Delta z}{A'} \end{aligned} \quad (19)$$

Satter(13) defined the percent heat loss as the heat loss divided by the usable heat content of the inlet steam. The usable heat content was defined as the enthalpy of the inlet steam minus the enthalpy of the liquid at the surface temperature of the earth. For the injection of a saturated steam, the mathematical equation would be:

$$\text{Percent heat loss} = \frac{\left[(y_o - y(z,t))H_v + \frac{zg}{Jg_c} \right] 100}{H_L + y_o H_v - (H_L)_{@T_E=a}} \quad (20)$$

Squier et al(15) derived an analytic solution for the temperature profile of a water injection well. As they assumed an infinite heat transfer coefficient between the fluid and the formation, their solution would be limited to problems involving injection down a casing.

THEORY

The equations stated in this section are derived in Appendix I, Sections A, B and C, and are subject to the following basic assumptions:

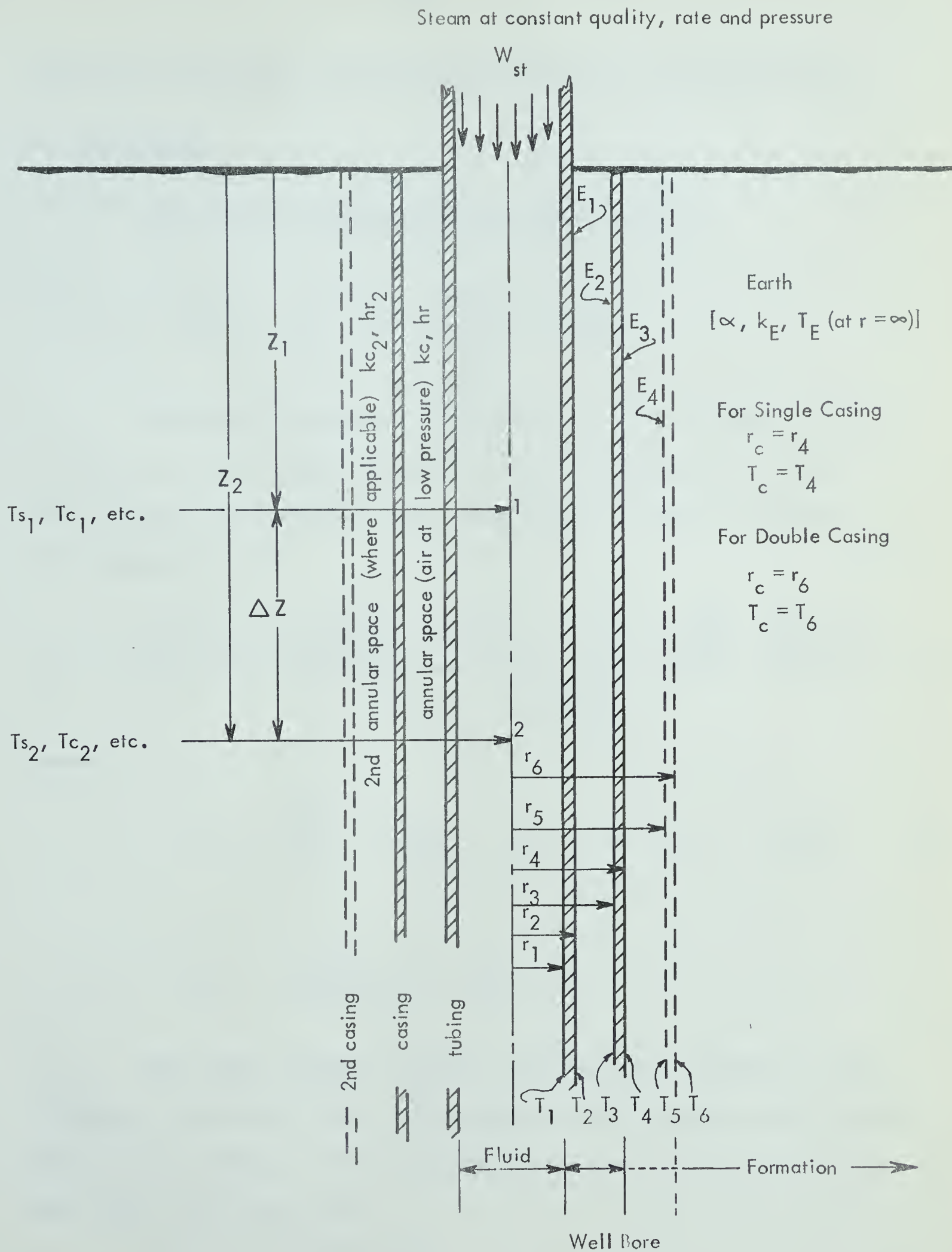
- 1) saturated steam is injected through the tubing at constant inlet conditions (quality, flowrate and temperature)
- 2) heat transfer in the wellbore is steady state, whereas heat transfer in the formation is unsteady state radial conduction.
- 3) physical and thermal properties of the formation are independent of depth and temperature.
- 4) the annulus contains air at a constant pressure.

The basic difference between this model and those proposed by Ramey(11) and Satter(13) is the inclusion of friction and kinetic energy effects. To obtain the mathematical model, the physical system was divided into three systems, resulting in three separate equations. The three systems were:

- a) the fluid
- b) the wellbore
- c) the formation

The heat flux across the system boundaries served as the parameter interrelating the three systems. The equations were derived assuming that the systems were uncoupled, whereby all the variables pertaining to another system were assumed to be known. The final solution was the result of successive

FIG. 1. DIAGRAM OF THE PHYSICAL SYSTEM





iterations until all systems were satisfied simultaneously.

a) The Fluid

The energy balance for the steam yields:

$$dH - \frac{g}{g_c} \frac{dz}{J} + \frac{udu}{Jg_c} = dq_w$$

Assuming the casing temperature to be a linear function of depth over the interval Δz , an equation describing the steam quality at the bottom of the interval may be derived (see Appendix I-A).

$$y_2 = \frac{\left[y_1 (H_{v_1} + y_1 f_1 v_{G_1}^2) - C_{sL} (T_{s_2} - T_{s_1}) + \left(\frac{g}{g_c J} - \frac{q_r}{W_{st}} \right) \Delta z \right]}{(H_{v_2} + y_2 f_1 v_{G_2}^2)} \quad (A7)$$

where

$$q_r = \frac{\pi k_E}{f(t)} \left[(T_{c_1} - T_{E_1}) + (T_{c_2} - T_{E_2}) \right] \quad (A4)$$

$$f_1 = \left(\frac{W_{st}}{3600 \pi r_1^2} \right)^2 \frac{1}{2g_c J}$$

The force balance relates the friction losses of the steam to a pressure drop. The equation describing the pressure drop for two phase, vertical steam flow down a circular pipe was taken from Perry(10).

It may be written as:

$$-\frac{dP}{dz} = \frac{f_G \rho_G u_G^2}{g_c r_1} \quad (A8)$$

where

$$f_G = \text{two phase friction factor} = f(R_{EG}, \gamma) \quad (A9)$$

(The graphical relationship, as presented in Perry(10) is included in Appendix III-A as Figure 14)

$$R_{EG} = \frac{2W_{st} \gamma}{\pi r_1^4 \mu_G}$$

$$\gamma = \frac{(1 - \gamma)W_{st}}{2\pi r_1 \rho_L} \quad (A10)$$

Equation A8 may be rewritten to yield:

$$\frac{dP}{v_G} = \lambda f_G \gamma^2 dz$$

where

$$\lambda = \frac{-1}{(12.3600)^2} \frac{W_{st}^2}{g_c \pi^2 r_1^5}$$

To eliminate the need for interpolation and extrapolation routines necessary if the graph for f_G were to be applied directly, simplifying assumptions were made. The liquid density and vapor viscosity were assumed constant, and f_G was plotted versus quality for various inlet conditions (See Figure 15 and Figure 17 in Appendix III-A). A straight line fit was generally sufficiently accurate over the anticipated

quality ranges. At high injection rates the coefficients for the line were relatively independent of superficial velocity (see Figure 17). Thus:

$$f_G = a_y + b_y y \quad (A14)$$

Keenan and Keyes(8) present mathematical relationships between vapor pressure and temperature, and vapor volume and temperature for saturated steam.

$$P = \phi(T_s) \quad (A15)$$

$$V_G = \psi(T_s, \phi(T_s)) \quad (A16)$$

employing equations A11, A14, A15 and A16, it was possible to derive:

$$\Delta z = z_2 - z_1 = \frac{I_T}{\lambda (a_y + b_y \bar{y}) \bar{y}^2} \quad (A18)$$

where

$$I_T = \int_{T_{s1}}^{T_{s2}} \frac{\phi'(T_s)}{\psi(T_s, \phi(T_s))} dT_s \quad (A19)$$

$$\bar{y} = \frac{y_1 + y_2}{2}$$

The substitution of equation A7 into equation A18 results in one implicit equation in z_2 and T_{s2} . Upon specification of T_{s2} , z_2 may be obtained by trial and error, using Newton's method for convergence.

b) The Wellbore

Assuming steady state heat transfer in the wellbore, the heat transferred through any part of the wellbore must be equal. Based on this criteria, an overall heat transfer coefficient[†] based on the outside casing area may be defined.

$$\frac{1}{U_o} = \frac{r_4}{r_1 h_s} + \frac{r_4 \ln \frac{r_2}{r_1}}{k_1} + \frac{r_4}{r_2 h_r + \frac{k_c}{\ln \frac{r_3}{r_2}}} + \frac{r_4 \ln \frac{r_4}{r_3}}{k_2} \quad (B3)$$

where k_c is defined by equation B5 in Appendix I-B. Equation B3 may be rewritten as :

$$U_o = \frac{1}{C_3} \left(1 - \frac{1}{C_3 \cdot C_4 (T_2 + T_3) (T_2^2 + T_3^2) + C_3 \cdot C_5 k_c + 1} \right) \quad (B4)$$

where

$$C_3 = \frac{r_4}{r_1 h_s} + \frac{r_4 \ln \frac{r_2}{r_1}}{k_1} + \frac{r_4 \ln \frac{r_4}{r_3}}{k_2}$$

$$C_4 = \frac{\sigma r_2}{r_4} \left[\frac{1}{\frac{1}{\epsilon_1} + \frac{r_2}{r_3} \left(\frac{1}{\epsilon_2} - 1 \right)} \right]$$

$$C_5 = \frac{1}{r_4 \ln \frac{r_3}{r_2}}$$

† Although only the single casing case is treated in this section, single and double casing relationships are derived in Appendix I-E

Equating the heat transferred through the wellbore to the heat entering the formation and solving for the outside casing temperature yields:

$$F_T = T_C - T_S + \frac{k_E (T_S - T_E)}{k_E + U_O r_C f(t)} = 0 \quad (B7)$$

Assuming $T_2 = T_S - \Delta_t$ and $T_3 = T_C + \Delta_c$, where Δ_t and Δ_c are independent of T_C ; the substitution of equation B4 into equation B7 yields one implicit equation in T_C . The casing temperature (T_C) may be determined by trial and error, employing Newton's method for convergence. Thus:

$$T_C^{(n+1)} = T_C^{(n)} - \frac{F_T^{(n)}}{\left(\frac{dF_T}{dT_C}\right)^{(n)}}$$

After a value has been obtained for T_C which satisfies equation B7, U_O may be calculated using equation B4.

c) The Formation

Assuming a time independent steam temperature, the partial differential equation describing radial heat conduction in cylindrical coordinates may be written as:

$$\frac{\partial^2 T^*}{\partial r^2} + \frac{1}{r} \frac{\partial T^*}{\partial r} = \frac{1}{\alpha} \frac{\partial T^*}{\partial t} \quad (C4)$$

where

$$T^* = T - T_S$$

Carslaw and Jaeger(4) provide an analytical solution for the partial differential equation, subject to the following conditions:

Initial condition: $T^*(r \geq r_c, 0) = T_E - T_S = T_O^*$

Boundary conditions: 1) T^* is finite at $r = \infty$

$$2) \left. \frac{\partial T^*}{\partial r} \right|_{r=r_c} = \frac{U_O}{k_E} T^* \Big|_{r=r_c} \quad (C5)$$

Assumption: U_O is not a function of time.

Since only the heat flux at the wellbore interface is required, the solution in Carslaw and Jaeger may be simplified to yield:

$$\left. \frac{\partial T}{\partial r} \right|_{r=r_c} = \frac{4U_T^2 T_O^*}{\pi^2 r_c} \int_0^\infty \left[\frac{e^{-t_D \omega^2}}{[\omega J_1(\omega) + U_T J_O(\omega)]^2 + [\omega Y_1(\omega) + U_T Y_O(\omega)]^2} \right] \frac{d\omega}{\omega} \quad (C7)$$

From the definition of $f(t)$, the following relationship may be derived:

$$f(t) = \frac{T_E - T_S}{r_c \left. \frac{\partial T}{\partial r} \right|_{r=r_c}} - \frac{1}{U_T} \quad (C10)$$

The evaluation of the integral in equation C7, necessary in the calculation of $f(t)$, is described in Appendix I-C

Summarizing the equations developed in this section, it is possible to describe the physical system with three interrelated equations.

$$1) \quad z = F_1(T_s, T_c, f(t), z) \quad (\text{equation A18})$$

$$2) \quad T_c = F_2(T_s, f(t), T_c, z) \quad (\text{equation B7})$$

$$3) \quad f(t) = F_3(T_c, T_s, t, z) \quad (\text{equation C10})$$

As there are five variables (T_s , T_c , $f(t)$, z and t) and only three equations, two variables may be specified. Due to the nature of the equations, the specification of T_s and t would lead to the simplest calculations. After all variables have been determined, other parameters such as quality and heat losses may readily be calculated.

DISCUSSION

a) Discussion of the Model

The basic assumptions leading to the mathematical model are restrictive, but often necessary. Although it may be possible to remove them by a completely numerical solution, the complexity of the problem increases by such an extent that any gain from a better model is soon lost to round-off errors. It is believed that a comprehensive solution circumventing the assumption of steady state heat transfer is difficult, if not impossible. The complicated boundary conditions, added to temperature dependent radiation and convection terms, tend to rule out an unsteady state analytic solution. The number of grid points for a numerical solution are limited, as radiation and natural convection make it difficult to subdivide the annulus. Comparing the heat absorbed by the wellbore to the heat transferred through the wellbore, it is conceivable that the error introduced by this assumption is negligible, except for very short times.

Constant inlet conditions were necessary insofar as they guaranteed a relatively constant temperature profile for the steam, a stipulation imposed by the analytic solution for $f(t)$. No attempt has been made to obtain or develop an analytic solution for $f(t)$ incorporating a time dependent steam temperature, but it is unlikely that such a solution exists.

The problem was limited to a solution for saturated steam injection, as existing solutions(11,13,15) for single phase fluid injection are believed to be adequate. For single phase fluids, the friction loss would affect the heat loss to the extent that pressure would affect the fluid enthalpy. Since the pressure drop for a single phase fluid is considerably lower than for a two-phase fluid, the overall effect of friction is believed to be minor.

The assumptions of constant annular pressure and depth independent thermal and physical properties of the formation could readily be removed by stratification of the wellbore and the formation.

The additional assumption of a constant steam temperature, as made by Ramey(11) and Satter(13) eliminates one equation (A18) and one variable (T_g). The assumption, resulting from the neglect of friction losses, is necessarily poor for flow involving two phases and high velocities. Economic considerations dictate high injection rates, resulting in high velocities, which subsequently cause sizeable friction losses. Field measurements(3) verify this hypothesis by presenting a pressure drop of 100 psi in 1600 ft. of tubing (see Figure 4). Neglecting friction losses, any injection velocity becomes theoretically possible; creating a possibility of performing heat loss and quality calculations for an injection velocity that is impossible due to excessive friction losses.

The mathematical model is believed to be reasonably comprehensive and any difference between theoretical and actual results could presumably be traced to the empirical correlations. The correlation employed to determine friction losses is probably open to the most discussion, as a correlation that is basically unreliable(10) is subjected to extrapolations and interpolations. A straight line approximation for the friction factor versus quality relationship simplified the calculations, but presumably led to errors that were pronounced for qualities greater than 0.95 or less than 0.4 (see Figure 15 and Figure 17). Considering the correlation and the approximations utilized, it would be optimistic to expect a completely reliable temperature profile. But until complete experimental or field data are available to evaluate all possible two phase flow correlations, it is felt that more sophisticated correlations would magnify calculational difficulties without necessarily yielding better results. Whereas most other correlations (2,5,7,9,16) were derived empirically for the horizontal flow of oil and gas mixtures, the relationship employed was developed expressively for the type of flow encountered in two phase steam injection.

The approximations made in developing the $f(t)$ function (see Appendix I-C) were reasonable, as neither the change in the wellbore heat transfer coefficient nor the change in steam temperature (see Figure 3 and Figure 11) with time was severe.

b) Discussion of Results

The sample cases run on the computer yielded results that are believed to be reasonable and readily explainable.

A simplified computer program, neglecting frictional effects was employed in an attempt to duplicate results published by Satter(13). Figure 2 indicates that Satter calculated heat losses that were approximately four percent higher than those obtained from the simplified computer program; whereas, Figure 3 indicates a difference of approximately 12°F in the calculated casing temperatures. The reason for the disagreement is difficult to determine, as Satter presented incomplete data by neglecting to specify values for emissivities and annular pressure. He also neglected to specify the correlation employed to determine k_c and the assumptions made in calculating $f(t)$. At a flowrate of $3000 \text{ lb}_m/\text{hr}$ the effect of friction on the total heat loss is negligible, as shown on Figure 2. Increasing the flowrate to $5000 \text{ lb}_m/\text{hr}$, the neglect of friction yields heat losses that are five percent higher. The frictional effects reduce the steam temperature, subsequently reducing the heat losses from the wellbore. The effect of friction on the total heat loss is not pronounced, as temperature changes are insignificant at shallow depths. If friction is neglected, the curvature of the graphs in Figure 2 is solely the result of the temperature gradient in the earth.

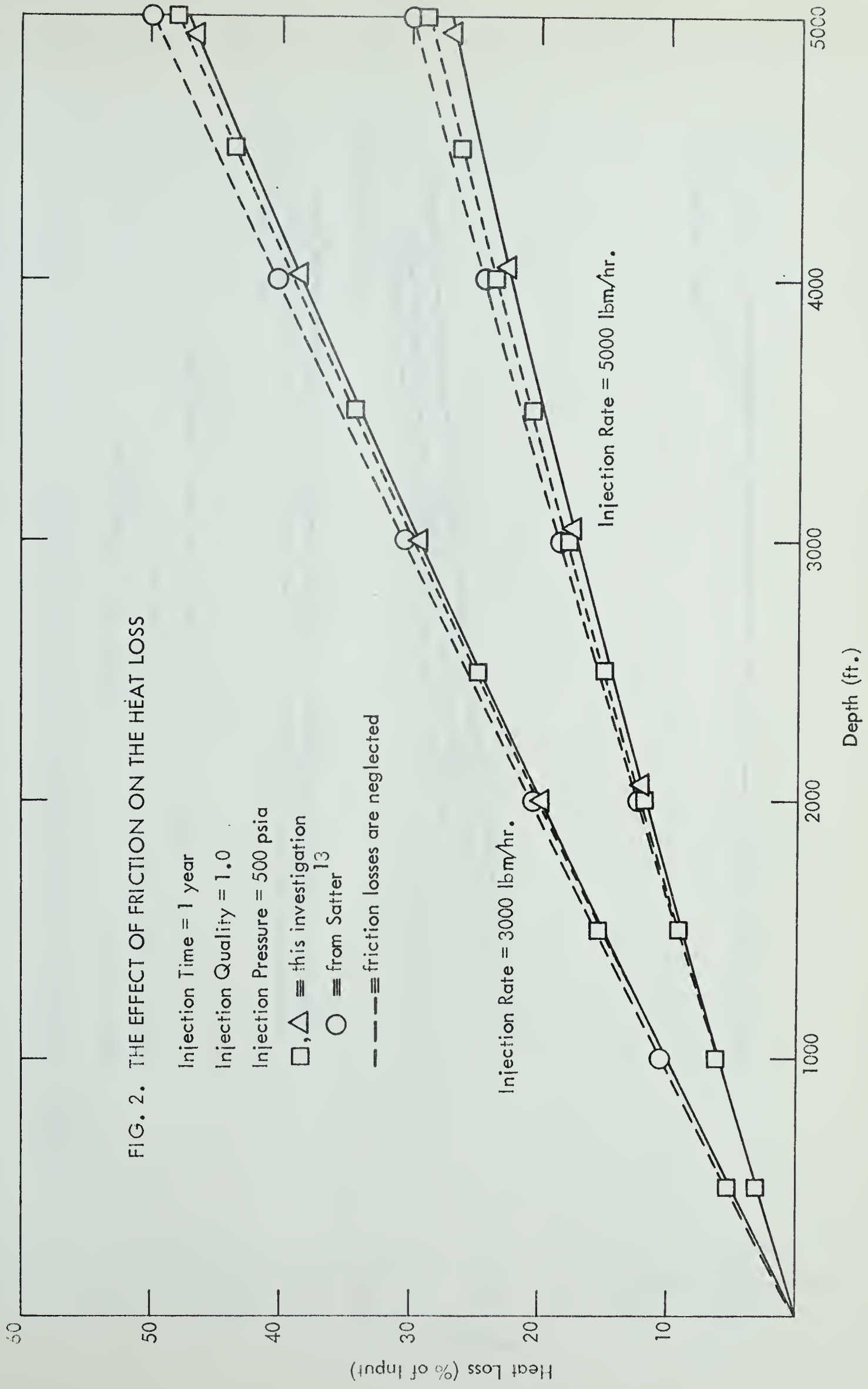
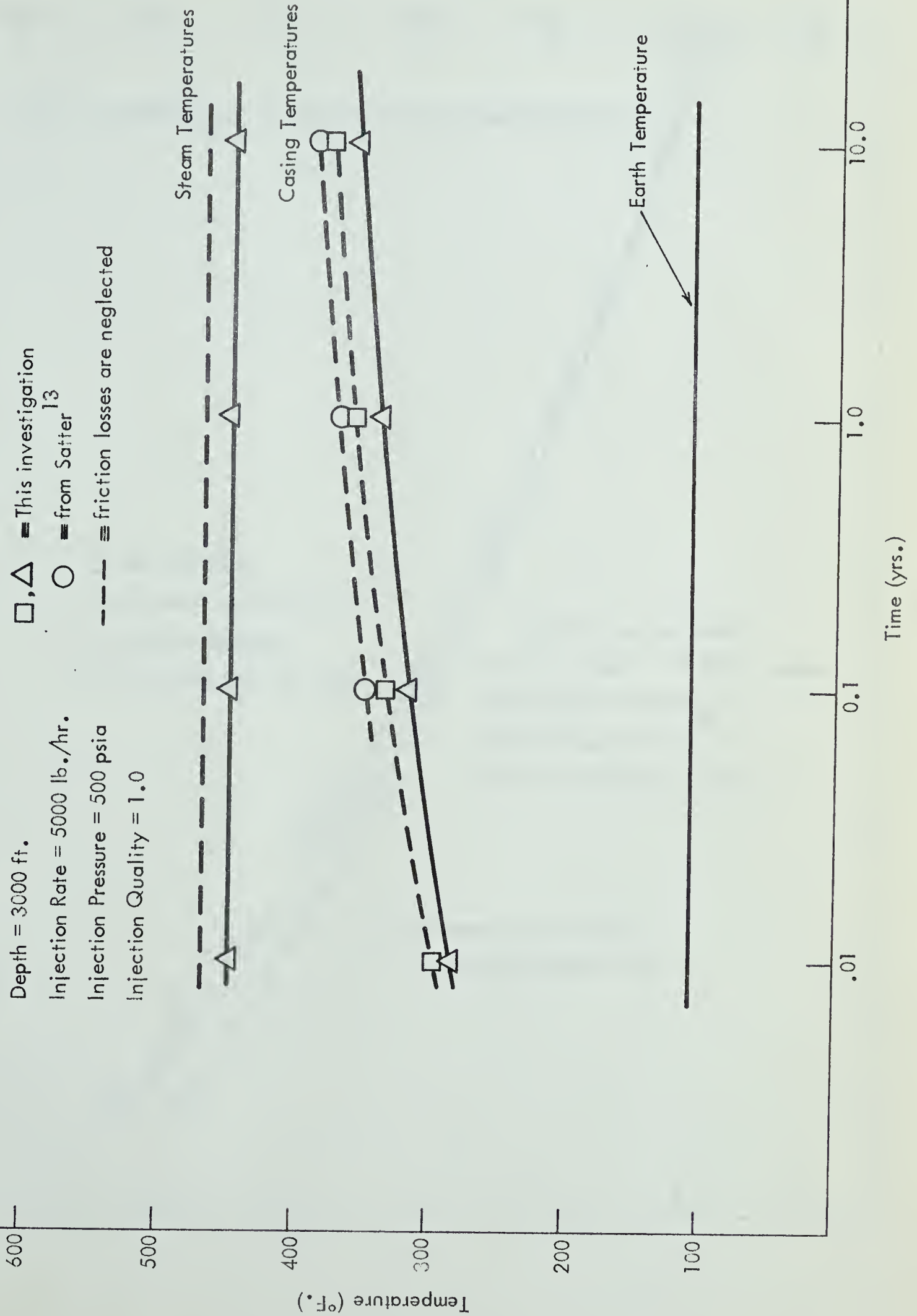


FIG. 3. THE EFFECT OF FRICTION ON THE CASING TEMPERATURE



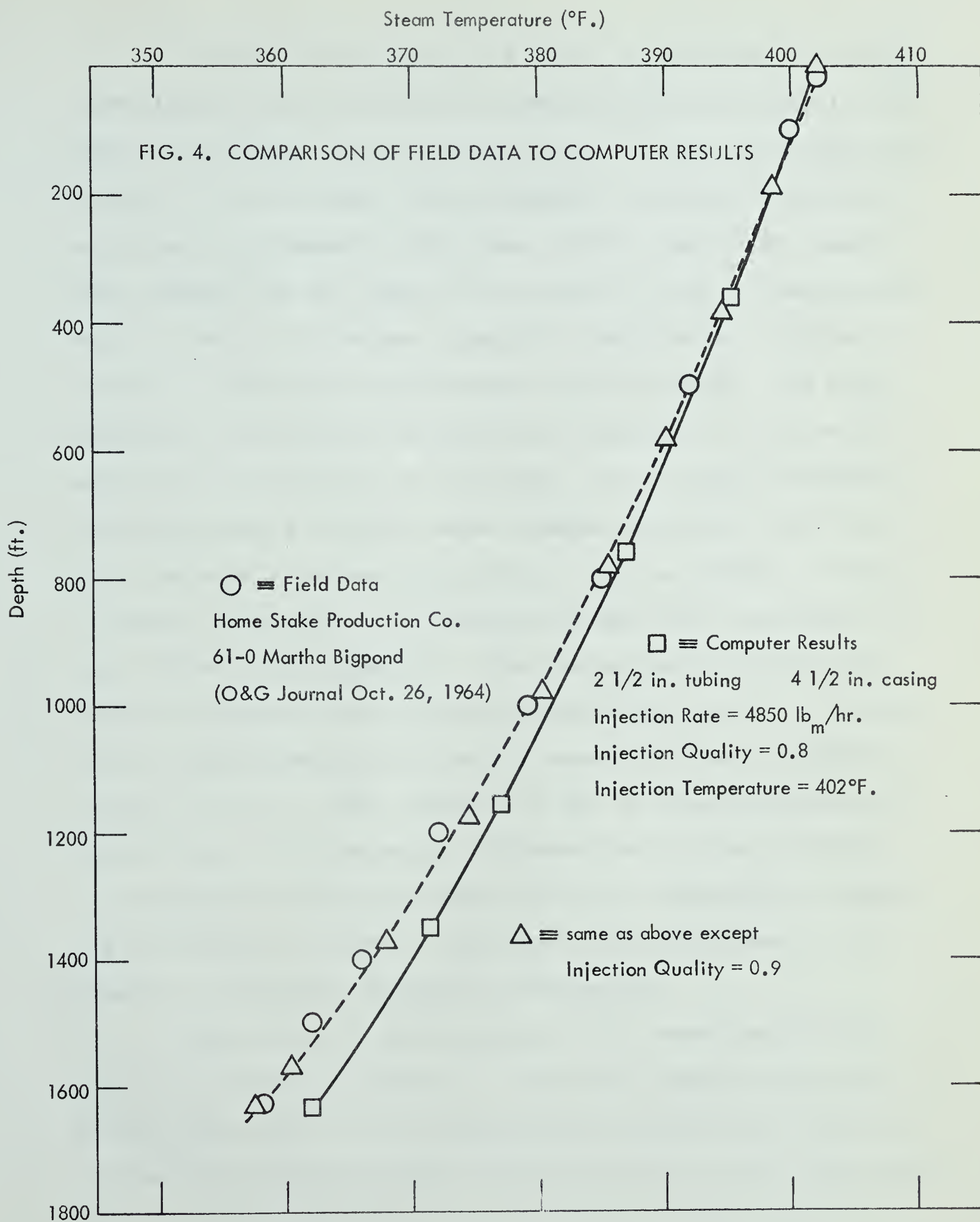
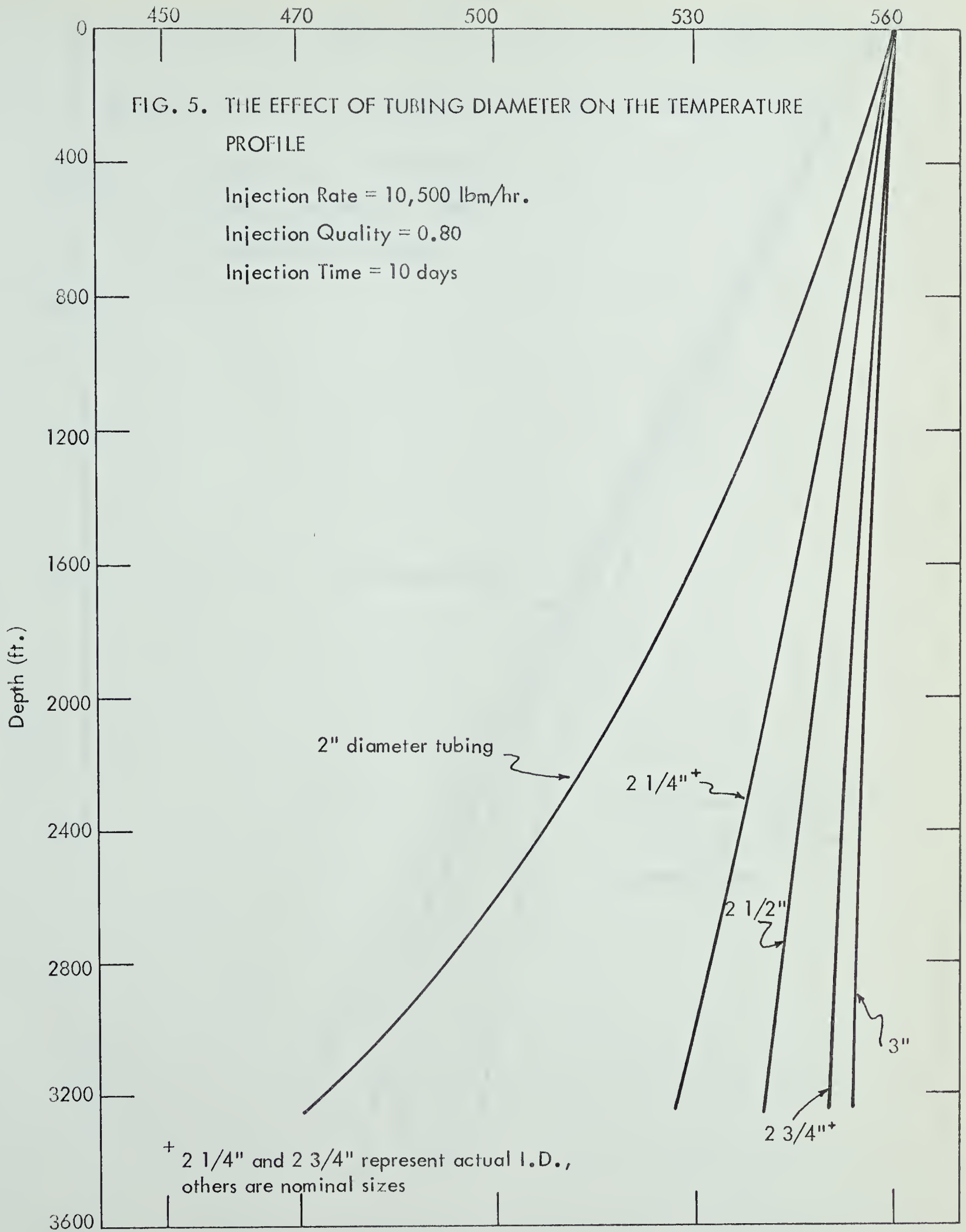


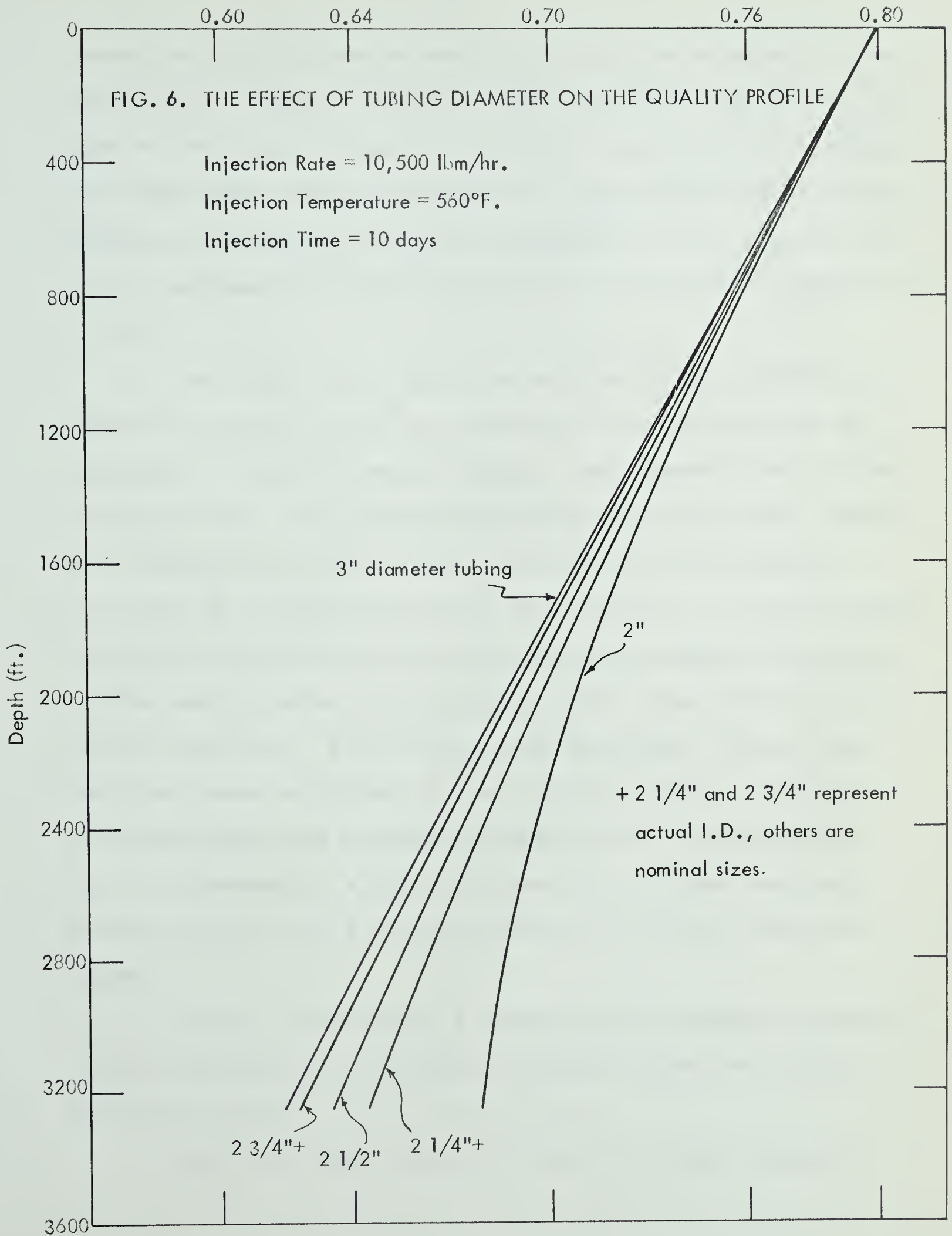
Figure 4 represents an attempt to evaluate the model, specifically the two phase flow relationship, by calculating a temperature profile and matching it to an observed temperature profile. All the known data (flowrate, geothermal gradient and injection pressure) were used, while reasonable values were assumed for the rest of the required data. The computer results for two different injection qualities are plotted in Figure 4, along with the measured field data(3). The close agreement obtained for an injection quality of 0.9 does not verify the reliability of the model, as too many parameters, including tubing diameter were assumed. However, the field data presents a temperature profile that is similar in shape to those calculated, which would indicate that the model is a step in the right direction. The temperature decrease per foot of wellbore increases from approximately 0.02 at the surface to approximately 0.04 at the sandface (for an injection quality of 0.9). This increase is due to lower temperatures, higher heats of evaporation and lower heat losses combining to increase the volume of vapor flowing, subsequently increasing the friction losses. Increased friction losses in turn result in increased temperature decrements.

The effect of tubing size on the steam temperature profile is shown in Figure 5. Since the pressure drop for single phase gas flow is approximately proportional to $1/r_1^5$, a temperature profile that is very sensitive to the fractional

Temperature (°F.)



Quality

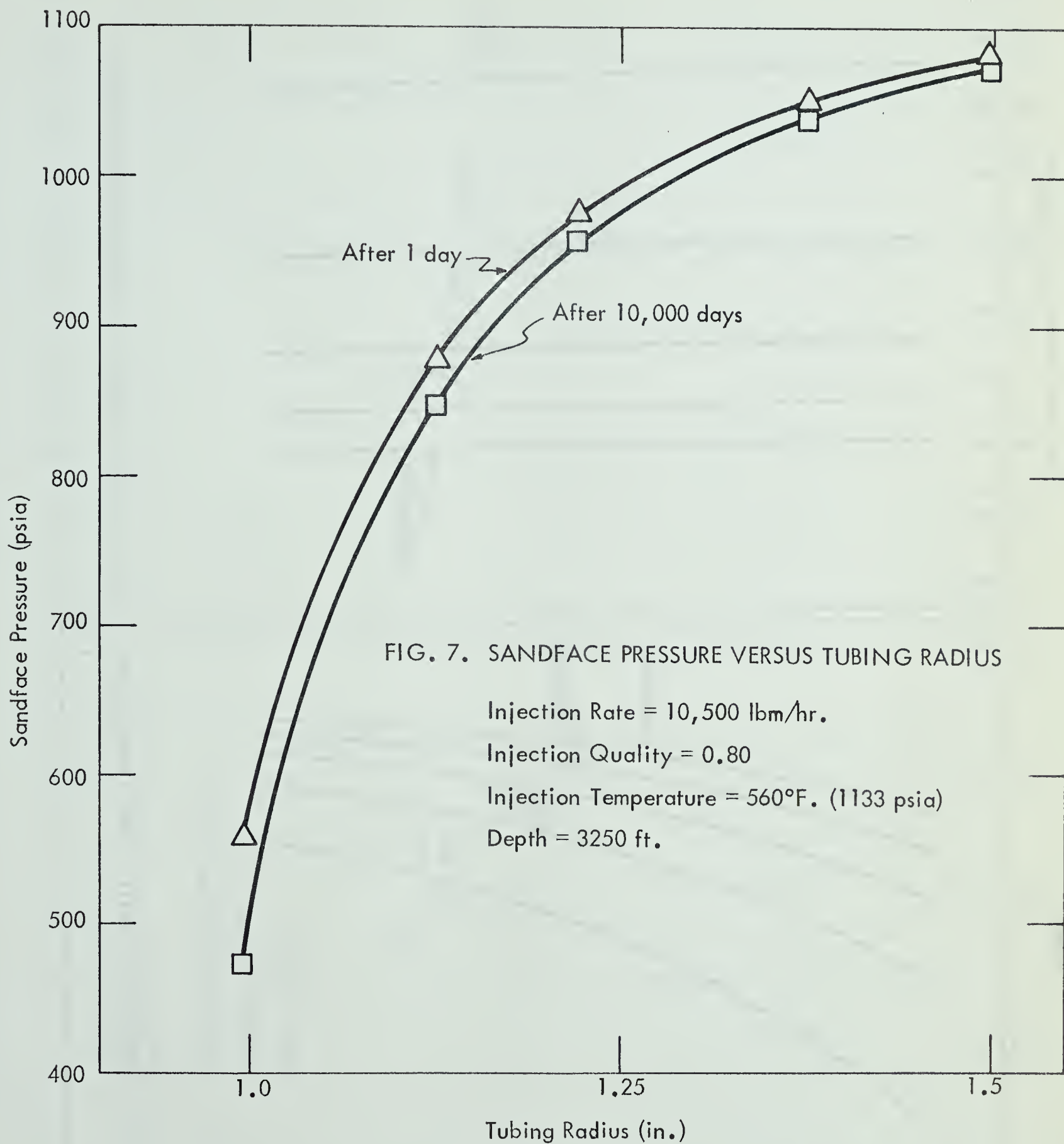


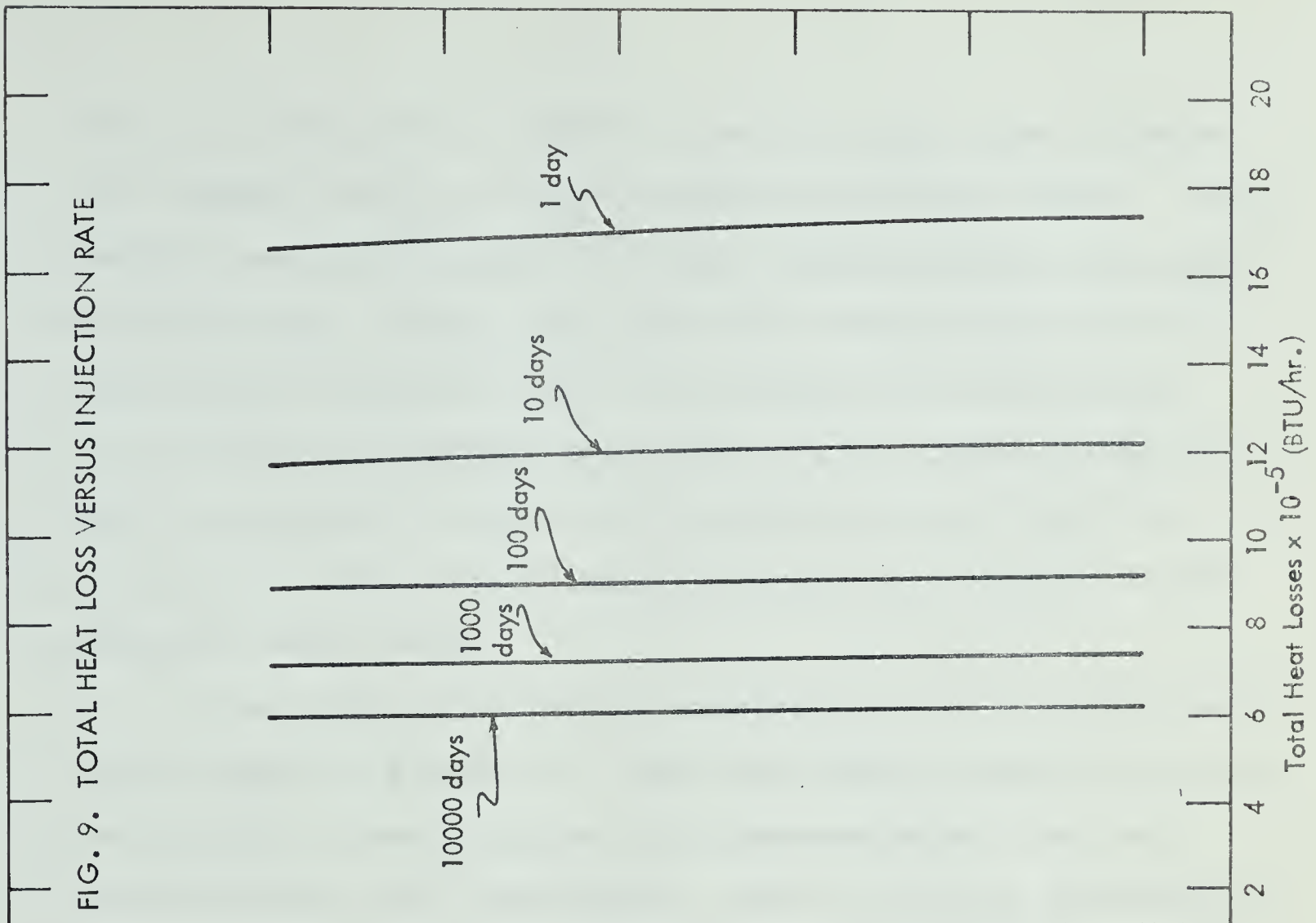
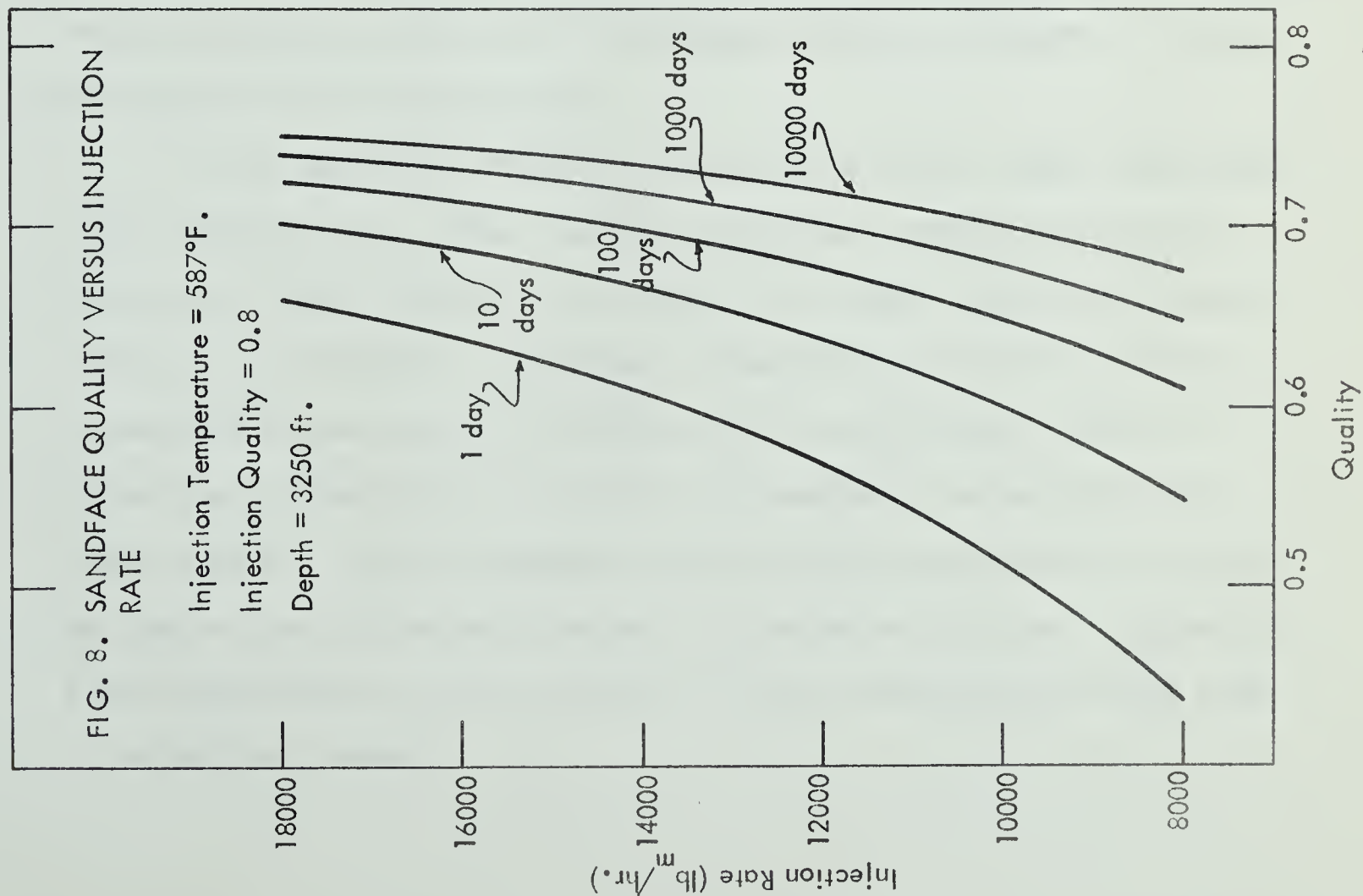
change in tubing diameter would similarly be expected for two phase flow. Figure 5 verifies this hypothesis as multiplication of the tubing diameter by 1.125 (from 2" to 2¼") reduces the temperature drop by 63 percent (from 90°F to 33°F); whereas, multiplication of the tubing diameter by 1.09 (from 2-3/4" to 3") decreases the temperature drop by 35 percent (from 10°F to 6½°F).

The effect of tubing size on the quality profile is shown in Figure 6. For high velocity flow, as would be encountered in small diameter tubing, steam temperature changes would be high. Thus, with increasing depths the heat losses will decrease and the heat of vaporization will increase, resulting in a lower fractional condensation or quality change. It would be possible to have the quality increase with depth if the loss of potential energy is larger than the heat loss to the formation. If friction were neglected, tubing size would not have any effect on the quality profile. Figure 6 indicates that this assumption could lead to considerable error by presenting a quality change of 0.12 for two-inch diameter tubing and a quality change of 0.18 for three-inch tubing.

Figure 7 represents a cross-plot of sandface pressure versus tubing radius and exemplifies the effect of tubing size on friction.

The total heat losses are lower at high injection

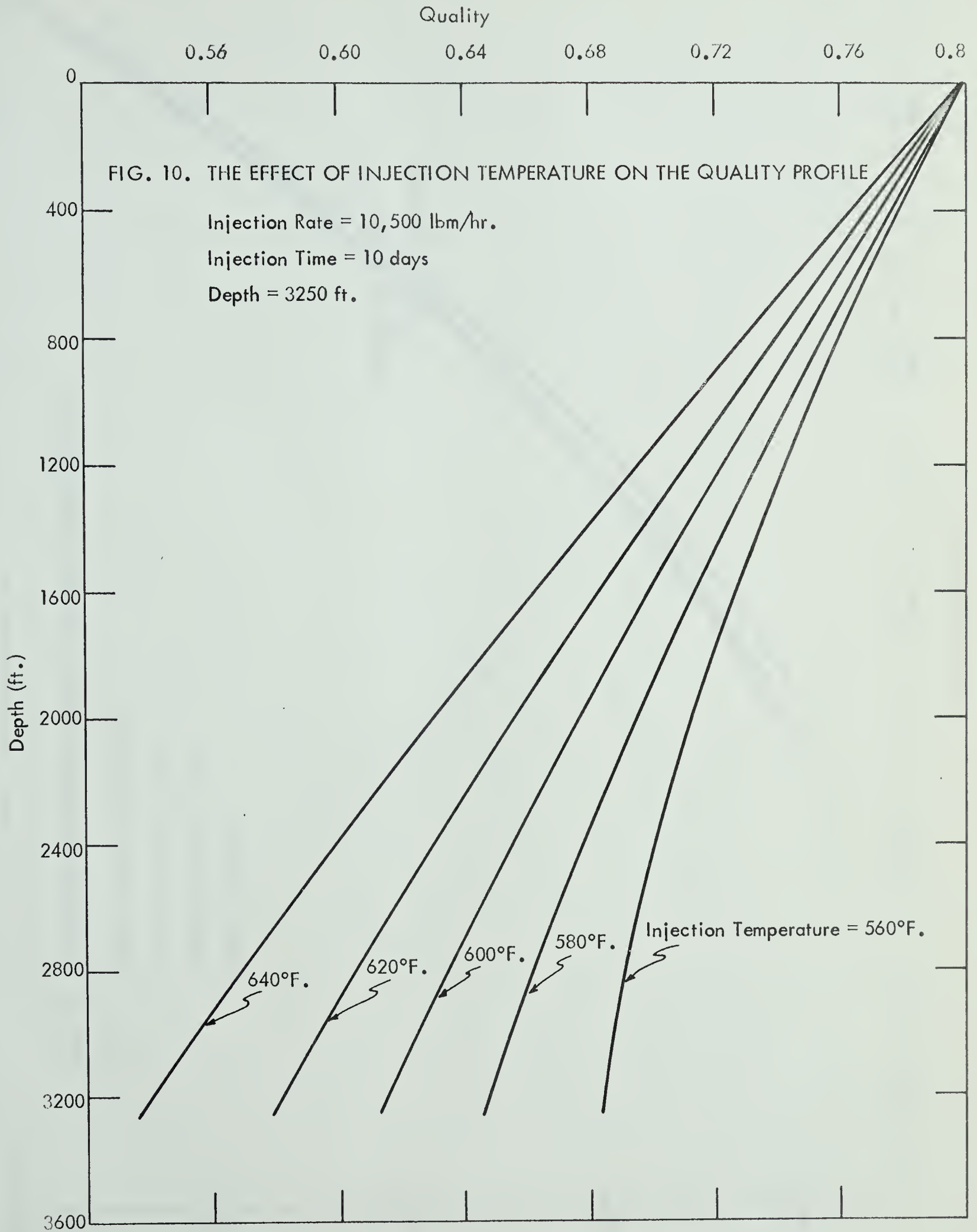


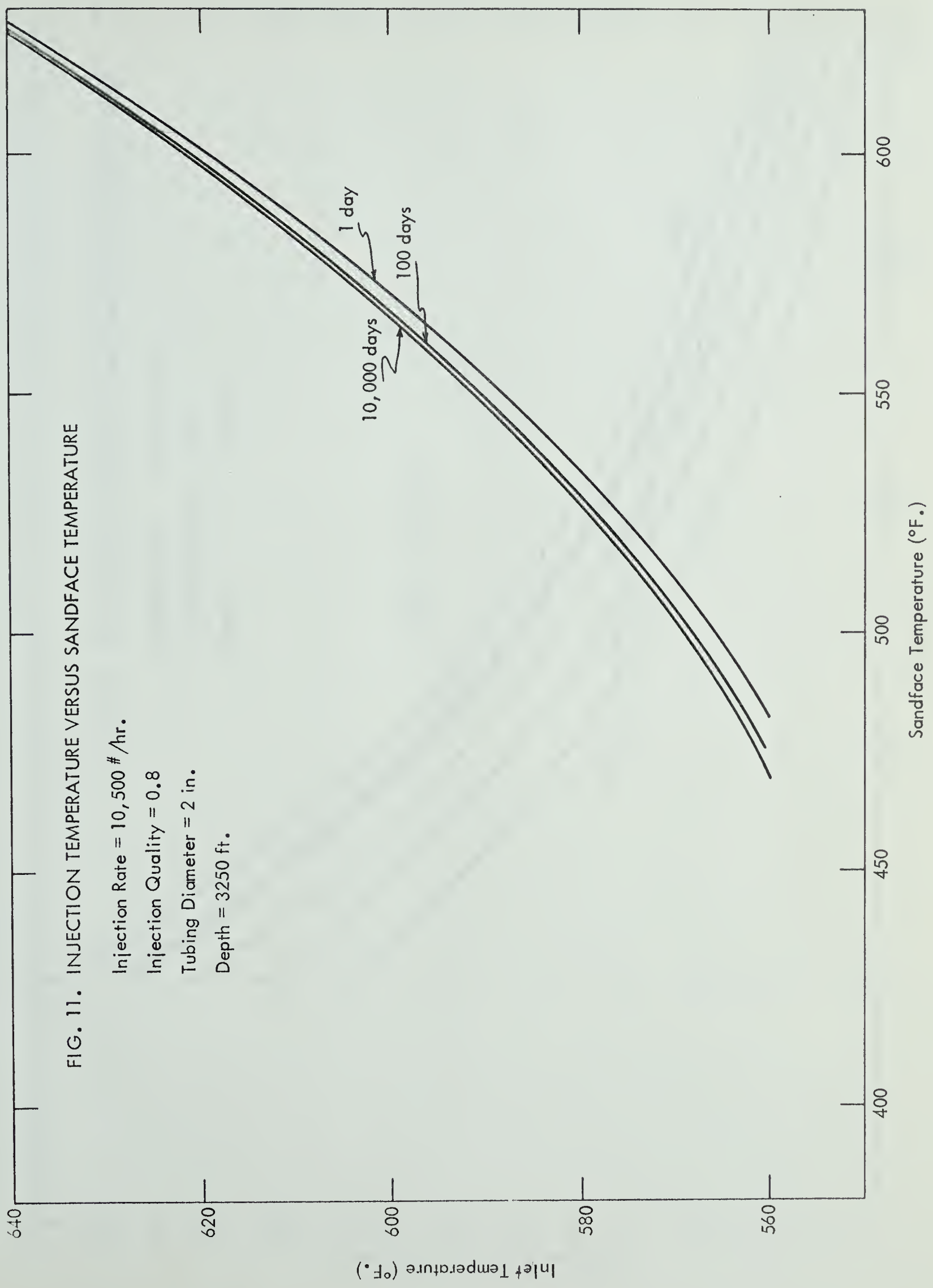


rates, as indicated in Figure 9, due to larger steam temperature changes arising out of increased frictional effects. The sandface quality is higher for high injection rates, since more available vapor, lower heat losses and larger enthalpies of vaporization combine to cause less fractional condensation. If the effect of friction was neglected, the curves in Figure 8 would be expected to approach an asymptote as the heat loss per pound of steam flowing becomes negligible compared to the potential energy loss.

The effect of injection temperature on the quality profile is shown in Figure 10. Since the heat of vaporization and the friction losses decrease with temperature and the heat losses increase with temperature, higher injection temperatures would be expected to yield increased quality changes. Figure 10 verifies this hypothesis.

The specific volume of saturated water vapor decreases with temperature. Thus, higher injection temperature would result in lower velocities, leading to lower friction losses. Figure 11 indicates a sandface temperature of 470°F for an injection temperature of 560°F after 10,000 days. For an injection temperature of 640°F the sandface temperature becomes 625°F. The bottomhole temperature decreases with time, as the volume of vapor flowing increases with time. Figure 11 also indicates that the change in steam temperature with time is relatively small.





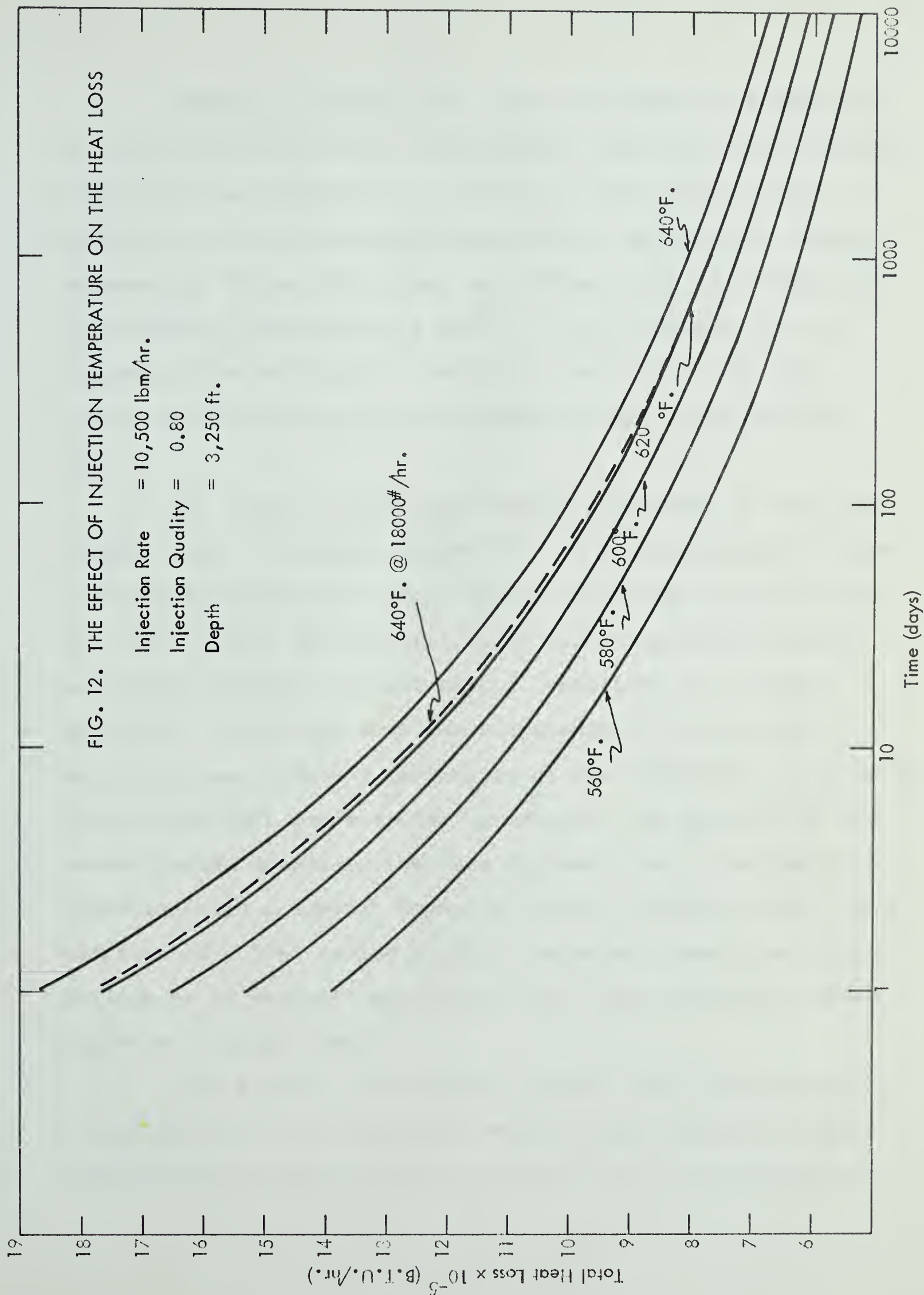


Figure 12 presents the effect of injection temperature on the rate of heat loss. As expected, the heat loss increases with injection temperature. Figure 12 also indicates the effect of time on the rate of heat loss. As the heat losses decrease by 27 percent in ten days (from 1 day to 10 days for an injection temperature of 560°F), it is apparent that an unsteady state solution is definitely required. All the curves should and appear to converge to zero at an infinite time.

The effect of changing various parameters on the rate of heat loss is shown in Figure 13. It is interesting to note that the addition of a second casing increases the heat losses to a low thermal conductivity formation. The second casing was added outside the first casing, resulting in a larger wellbore. It appears that the resistance of the annulus is slightly lower than the resistance of the formation it replaces. For high thermal conductivity formations, the addition of the second casing decreases the rate of heat loss. Considering a formation with a thermal conductivity of $1.4 \text{ Btu/hr.ft.}^{\circ}\text{F}$, the addition of a $10\frac{1}{2}$ " casing reduced the rate of heat loss after 10 days by 20 percent (see Figure 13). The reduction is even larger at shorter times.

From Figure 13 it becomes evident that radiation in the annulus is a very important mode of heat transfer, since decreasing the tubing emissivity from 0.8 to 0.2 reduced the

FIG. 13. THE EFFECT OF PARAMETER VARIATIONS ON HEAT LOSS

Injection Rate = 10,500 #/hr.

Injection Quality = 0.8

Injection Temperature = 560°F.

Depth = 3250 ft.

Second Casing Diameter = 10 1/2 in.

$K_E = 1.4 \frac{\text{BTU}}{\text{hr ft}^\circ\text{F.}}$
 $E_1 = 0.8$

$K_E = 0.4 \frac{\text{BTU}}{\text{hr ft}^\circ\text{F.}}$
 $E_1 = 0.8$

2 casings $K_E = 1.4$
 $E_1 = E_2 = E_3 = E_4 = 0.8$

2 casings: $E_1 = E_2 = E_3 = E_4 = 0.8$
 $K_E = 0.4$

$K_E = 0.4 \frac{\text{BTU}}{\text{hr ft}^\circ\text{F.}}$
 $E_1 = 0.2$

Total Heat Loss $\times 10^{-5}$ (BTU/hr.)

Time (days)

rate of heat loss by 30 percent (after 10 days). Reduction of the earth thermal conductivity from 1.4 Btu/hr.ft.^{°F} to 0.4 Btu/hr.ft.^{°F} decreased the rate of heat loss by 47 percent (after 10 days). Thus, the earth thermal conductivity and the emissivities become important parameters in determining a representative rate of heat loss from the wellbore.

The discussion of the results may be summarized.

1) Higher velocities resulted in higher friction losses.

2) Higher friction losses resulted in lower steam temperatures.

3) Lower steam temperatures resulted in lower heat losses, higher heats of evaporation and higher friction losses.

4) Lower heat losses resulted in smaller quality changes.

5) Higher heats of evaporation resulted in smaller quality changes.

6) Smaller quality changes resulted in increased friction losses.

Due to the large number of variables involved, it was difficult to evaluate the effect of every variable on the results. Thus, the graphs presented in this discussion are meant to be illustrative only, and to apply the proposed model, the computer program would have to be run using the specific data applicable to the problem to be solved.

CONCLUSIONS

The following conclusions were made on the basis of the results of the sample computer runs.

1) Consideration of friction led to lower heat losses, but the effect was relatively minor; whereas, the quality and the temperature profiles were greatly affected by friction.

2) The rate of heat loss may be reduced by the addition of a second casing or the reduction of the emissivities. The effectiveness of the addition of a second casing larger than the first casing, is governed by the thermal conductivity of the earth. If the thermal conductivity is low, the second casing may increase the rate of heat loss.

3) The earth thermal conductivity is a very important parameter in the determination of the rate of heat loss.

4) The shapes of the calculated temperature profiles were comparable to an observed temperature profile.

5) The effect of the time on the steam temperature and the overall wellbore heat transfer coefficient was relatively minor.

RECOMMENDATIONS

The assumption of a constant wellbore steam temperature is recommended for cases where only the rate of heat loss is desired. To ensure that friction losses are not excessive, a rough check on the possible friction losses would be advisable. The assumption would simplify the calculations considerably, and the error introduced would act as an inherent safety factor. To calculate representative heat losses, care must be exercised in arriving at a value for the thermal conductivity of the earth. If conductivity data is available as a function of depth, and considerable variations are indicated, it might be advantageous to divide the formation into strata of constant physical properties. For situations involving small steam and earth temperature changes with depth, and the total heat loss is the desired result, an average value for the thermal conductivity is believed to yield sufficient accuracy.

To calculate reasonably reliable values for the sandface pressure and quality, the effect of friction must be considered. Due to rapid error propagation resulting from the relationship between pressure and friction losses, a very good two phase flow correlation is required to obtain reliable sandface pressures. Until a correlation is derived that has been substantiated under conditions similar to field conditions, it is recommended to employ caution in using calculated sandface

temperatures, as the error could conceivably be considerable.

The computer program as it is included in Appendix II, could readily be modified to account for more resistances in the wellbore heat transfer coefficient and more elaborate relationships between air properties and temperature. These changes are not recommended, as it is believed that the small gain in accuracy will not justify the increased computational time. If at any time the two phase flow correlation is substantiated, it may be advantageous to employ a higher order polynomial for the relationship between friction factor and quality.

Of the means of decreasing the heat losses from the wellbore, the reduction of the emissivities is recommended above the addition of a second casing. Whereas, the reduction of the emissivities always reduces the heat losses, the addition of a second casing (larger than the first casing) may increase the heat losses. The extent and under which circumstances the emissivities are to be reduced becomes a matter of economics.

NOMENCLATURE

C_{P_a}	=	heat capacity of the annulus air (Btu/lb _m ^{°F})
C_{sL}	=	dH/dT of water at its bubble point (Btu/lb _m ^{°F})
f_G	=	two phase friction factor as correlated in Figure 14
$f(t)$	=	time function as defined in the Literature Review
Gr	=	Grashof number
H	=	enthalpy (Btu/lb _m)
h_r	=	radiation heat transfer coefficient as defined by equation B2
h_s	=	film coefficient on inside of tubing (Btu/hr.ft. ² °F)
H_v	=	enthalpy of vaporization (Btu/lb _m)
I_T	=	integral as defined by equation A19
I_{t_1}	=	integral as defined by equation C13
J	=	mechanical equivalent of heat (778 ft-lb _f /Btu)
k_a	=	thermal conductivity of the annulus air (Btu/hr.ft. °F)
k_c	=	effective thermal conductivity of the annular air space (Btu/hr.ft. °F)
k_E	=	thermal conductivity of the earth (Btu/hr.ft. °F)
k_1	=	thermal conductivity of the tubing (Btu/hr.ft. °F)
k_2	=	thermal conductivity of the casing (Btu/hr.ft. °F)
k_3	=	thermal conductivity of the second casing (Btu/hr.ft. °F)
M	=	molecular weight of air (28.9 lb _m /lb.mole)
P	=	steam pressure (psia)
P_a	=	air pressure in the annulus (psia)
Pr	=	Prandtl number

q_r	= average rate of heat loss (Btu/ft.hr.) as defined by equation A4
q_w	= heat gain of the steam (Btu/lb _m)
R	= Gas-law Constant (10.73 lb _f ft ³ /in ² lb.mole °R)
r_c	= radius of the wellbore (ft)
R_{EG}	= vapor phase Reynolds number
r_1	= inside radius of the tubing (ft)
r_2	= outside radius of the tubing (ft)
r_3	= inside radius of the casing (ft)
r_4	= outside radius of the casing (ft)
r_5	= inside radius of the second casing (ft)
r_6	= outside radius of the second casing (ft)
t	= time (hrs.)
T_c	= temperature of the outside of the last casing (°R)
t_D	= dimensionless time as defined by equation C9
T_E	= earth temperature (°R)
T_s	= steam temperature (°R)
T_{sk}	= steam temperature (°K)
T_i	= temperature (°R) at r_i
u	= velocity (ft/sec) or dummy variable
U_o	= overall heat transfer coefficient based on r_c (Btu/hr.ft. ² °F)
U_T	= dimensionless coefficient as defined by equation C8
V	= specific volume (ft ³ /lb _m)
W_{st}	= steam flowrate (lb _m /hr.)
y	= quality (fraction)
z	= depth (ft)

α	= thermal diffusivity of the earth ($\text{ft}^2/\text{hr.}$)
γ	= parameter as defined by equation A10
Δ_t	= temperature drop across the tubing ($^{\circ}\text{F}$)
Δ_c	= temperature drop across the casing ($^{\circ}\text{F}$)
Δ_{c_2}	= temperature drop across the second casing ($^{\circ}\text{F}$)
ρ	= density (lb_m/ft^3)
σ	= Stefan-Boltzman constant ($0.1713 \times 10^{-8} \text{ Btu/hr.ft.}^2 \text{ } ^{\circ}\text{R}^4$)
μ_a	= viscosity of the annulus air ($\text{lb}_m/\text{hr.ft.}$)
μ_G	= viscosity of saturated water vapor ($\text{lb}_m/\text{hr.ft.}$)
ϵ_1	= emissivity of the outside of the tubing
ϵ_2	= emissivity of the inside of the casing
ϵ_3	= emissivity of the outside of the casing
ϵ_4	= emissivity of the inside of the second casing

Note:

Any of the above symbols subscripted with:

- 1) a G, represents the vapor phase, e.g. ρ_G
- 2) an L, represents the liquid phase, e.g. ρ_L
- 3) a number, represents the point in the tubing, e.g. y_1 , or the annular space number, e.g. k_{c_2} , depending on context.

BIBLIOGRAPHY

1. Abromowitz, M. and Stegun, I.A. (ed): Handbook of Mathematical Functions, U.S. Dept. of Commerce, Nat. Bureau of Standards, App. Math. Series 55 (1965).
2. Baker, O.: "Simultaneous Flow of Oil and Gas Mixtures", The Pipeline Engineer, (Feb. 1960).
3. Bleakly, W.B.: "Case Histories of Thermal Recovery Operations", Oil and Gas Jour., (Oct. 26, 1964) 62, No. 43, 123.
4. Carslaw, H.S. and Jaeger, J.C.: Conduction of Heat in Solids, Second Ed., Oxford U. Press, Amen House, London (1959).
5. Chavez, J.A.: "New Correlation for Two Phase Pipeline Flow", Oil and Gas Jour., (Aug. 24, 1959) 57, No. 35, 100.
6. Groeber, H., Erk, S. and Grigull, U.: Fundamentals of Heat Transfer, Third Ed., McGraw-Hill Book Co., Inc., N.Y. (1961).
7. Huntington, R.L. and White, P.D.: "Horizontal Co-Current Two Phase Flow of Fluids in Pipelines", Petroleum Engineer (Aug. 1955) 27, No. 9, D40.
8. Keenan, J.H. and Keyes, F.G.: Thermodynamic Properties of Steam, John Wiley and Sons, Inc., N.Y. (1962).
9. Lockhart, R.W. and Martinelli, R.C.: "Proposed Correlation of Data for Iso-thermal Two Phase Two Component Flow in Pipes", Chemical Eng. Progress (Jan. 1949) 45, No. 1, 39.
10. Perry, J.H. (ed): Chemical Engineers Handbook, Fourth Ed., McGraw-Hill Book Co., Inc., N.Y., (1963).
11. Ramey, H.J., Jr.: "Wellbore Heat Transmission", Journal of Petroleum Technology, (April, 1962) 427.
12. Rohsenow, W.M. and Choi, H.Y.: Heat, Mass and Momentum Transfer, Prentice Hall Inc., Englewood Cliffs, N.J. (1963).
13. Satter, A., Jr.: "Heat Losses During Flow of Steam Down a Wellbore", SPE paper 1071, presented at Rocky Mountain Regional Meeting of the Society of Petroleum Engineers of AIME, Billings, Montana, June 10-11, 1965.

14. Smith, J.M. and Van Ness, H.C.: Introduction to Chemical Engineering Thermodynamics, Second Ed., McGraw-Hill Book Co., Inc., N.Y. (1959).
15. Squier, D.P., Smith D.D. and Dougherty, E.L.: "Calculated Temperature Behavior of Hot-Water Injection Wells", Journal of Petroleum Technology, (April, 1962) 436.
16. Yocum, B.T.: "A Mathematical Study of Two Phase Flow in Well Flow Lines", The Petroleum Engineer, (Nov., 1959 and Jan., 1960) 31, No. 12, B40 and 32, No. 1, B59.

A P P E N D I X I
DERIVATION OF EQUATIONS

I-A. THE FLUID

(Calculation of Quality and Depth)

Writing a differential energy balance(14) for the steam flowing down the tubing yields:

$$dH - \frac{g}{g_c} \frac{dz}{J} + \frac{u du}{J g_c} = dq_w - \frac{dW_s}{J} \quad (A1)$$

As there is no shaft work, $dW_s = 0$.

Defining $\overline{q_r}$ as the rate of heat loss per foot of pipe yields:

$$dq_w = - \frac{\overline{q_r}}{W_{st}} dz$$

Thus, equation A1 reduces to:

$$dH + \frac{u du}{J g_c} = \left(\frac{g}{g_c} \frac{1}{J} - \frac{\overline{q_r}}{W_{st}} \right) dz \quad (A2)$$

Assuming a depth interval small enough to assume linear casing and earth temperature profiles, and integrating equation A2 from point 1 to point 2 yields:

$$H_2 - H_1 + \frac{1}{2Jg_c} (u_2^2 - u_1^2) = \left(\frac{g}{g_c} \frac{1}{J} - \frac{\overline{q_r}}{W_{st}} \right) (z_2 - z_1) \quad (A3)$$

$f(t)$ is defined to make the following equation correct(11)

$$\overline{q_r} dz = \frac{2\pi k_E (T_c - T_E)}{f(t)} dz$$

Thus if:

$$T_E = a + bz$$

and

$$T_C = e + fz$$

$$q_r = \frac{1}{z_2 - z_1} \int_{z_1}^{z_2} \frac{2\pi k_E}{f(t)} (e + fz - a - bz) dz$$

Defining:

$$T_{C_1} = e + fz_1 \quad T_{E_1} = a + bz_1$$

$$T_{C_2} = e + fz_2 \quad T_{E_2} = a + bz_2$$

and assuming $f(t)$ is constant over the interval z_1 to z_2 yields:

$$q_r = \frac{\pi k_E}{f(t)} \left[(T_{C_1} - T_{E_1}) + (T_{C_2} + T_{E_2}) \right] \quad (A4)$$

If the vapor phase may be assumed to be the principal factor in determining kinetic energy effects, obtain:

$$W_G = Y W_{st} = \pi r_1^2 \rho_G u_G 3600$$

and

$$u = u_G = \frac{W_{st}}{\pi r_1^2 \cdot 3600} Y v_G$$

The evaluation of the kinetic energy contribution leads to:

$$\frac{1}{2g_c J} (u_2^2 - u_1^2) = f_1 (y_2^2 v_{G_2}^2 - y_1 v_{G_1}^2) \quad (A5)$$

where

$$f_1 = \frac{1}{2g_c J} \left(\frac{W_{st}}{\pi r_1^2 3600} \right)^2$$

Defining H_o as a reference enthalpy, can write:

$$H_2 = H_o + \int_{T_o}^{T_{s1}} C_{sL} dT_s + \int_{T_{s1}}^{T_{s2}} C_{sL} dT_s + H_{v2} y_2$$

$$H_1 = H_o + \int_{T_o}^{T_{s1}} C_{sL} dT_s + H_{v1} y_1$$

Thus:

$$H_2 - H_1 = \int_{T_{s1}}^{T_{s2}} C_{sL} dT_s + H_{v2} y_2 - H_{v1} y_1$$

For reasonable temperature changes, C_{sL} is nearly constant and thus obtain:

$$H_2 - H_1 = C_{sL} (T_{s2} - T_{s1}) + y_2 H_{v2} - y_1 H_{v1} \quad (A6)$$

Substituting equations A5 and A6 into equation A3 and solving for y_2 yields:

$$y_2 = \frac{\{y_1(H_{v1} + y_1 f_1 v_{G1}^2) - C_{sL}(T_{s2} - T_{s1}) + (\frac{g}{Jg_c} - \frac{q_r}{W_{st}})(z_2 - z_1)\}}{(H_{v2} + y_2 f_1 v_{G2}^2)} \quad (A7)$$

Equation A7 is implicit in y_2 . Since the kinetic energy contribution is generally quite small and a trial and error solution is required for the overall solution, the y_2 on the right

hand side will have little or no effect on the rate of convergence; whereas, an explicit equation would require the solution of a quadratic, which in turn would be time consuming.

The equation describing the pressure profile for the vertical flow of condensing steam down a circular pipe, is taken from Perry(10) p. 5 - 42.

It may be written as:

$$-\frac{dP}{dz} = \frac{4f_G \rho_G u_G^2}{2g_c 2r_1} \quad (A8)$$

where

$$f_G = F(R_{EG}, \gamma) \quad (A9)$$

(The graphical relationship for f_G was taken from Perry(10) p. 5 - 40, enlarged, and included in Appendix III-A as Figure 14.)

$$\gamma = \frac{(1 - y) W_{st}}{2\pi \rho_L r_1} \quad (A10)$$

$$u_G = \frac{y W_{st}}{\rho_G \pi r_1^2 3600}$$

$$R_{EG} = \frac{2\rho_G u_G r_1}{\mu_G} = \frac{2y W_{st}}{\pi r_1 \mu_G}$$

Equation A8 may be rewritten as:

$$\frac{dP}{v_G} = \lambda f_G y^2 dz \quad (A11)$$

where

$$\lambda = \left(\frac{W_{st}}{12 \cdot 3600 \cdot \pi} \right)^2 \frac{-1}{g_c r_1^5}$$

If the temperature change in the steam is not excessive, average values may be assumed for μ_G and ρ_L . Thus, for every run, a plot of quality versus two phase friction factor may be obtained, as:

$$R_{EG} = Y_a \cdot y$$

$$y = (1 - y) \cdot Y_b$$

where

$$Y_a = \frac{2W_{st}}{\pi r_1^2 \mu_G} \quad (A12)$$

$$Y_b = \frac{W_{st}}{2\pi r_1^2 \rho_L} \quad (A13)$$

It was found that for relatively high Y_a and Y_b , as would be encountered in a field case; a straight line yielded a reasonable fit for f_G vs y in the interval $0.3 < y < 0.95$ (see Figure 15 and Figure 17). The coordinates of this line were nearly independent of Y_a and Y_b (see Figure 17). Thus:

$$f_G = a_y + b_y \cdot y \quad (A14)$$

From Keenan and Keyes(8) p. 14 and p. 15 obtain:

$$P = \phi(T_s) ,$$

Therefore,

$$dP = \phi' (T_s) dT_s \quad (A15)$$

$$v_G = \psi(T_s, P) = \psi(T_s, \phi(T_s)) \quad (A16)$$

Substituting equations A14, A15 and A16 into equation A8, and integrating from point (1) to point (2) yields:

$$\int_{T_{s1}}^{T_{s2}} \frac{\phi' (T_s)}{\psi(T_s, \phi(T_s))} dT_s = \int_{z_1}^{z_2} (a_y + b_y y) y^2 dz \quad (A17)$$

If the change in quality over the interval z_1 to z_2 is not excessive, use of an average quality over the interval is believed to result in sufficient accuracy.

Thus:

$$\Delta z = z_2 - z_1 = \frac{I_T}{\lambda (a_y + b_y \bar{y}) \bar{y}^2} \quad (A18)$$

where

$$I_T = \int_{T_{s1}}^{T_{s2}} \frac{\phi' (T_s)}{\psi(T_s, \phi(T_s))} dT_s \quad (A19)$$

$$\bar{y} = \frac{y_1 + y_2}{2}$$

Equation A7 may be substituted into equation A18, resulting in one equation with two variables (T_{s2} and z_2). Specification of T_{s2} allows the determination of z_2 by a trial and error

solution of the implicit equation. Newton's method was applied for faster convergence.

$$F_z = \Delta z - \frac{I_T}{\lambda \bar{y}^2 (a_y + b_y \bar{y})} = 0 \quad (A20)$$

$$\Delta z^{(n+1)} = \Delta z^{(n)} - \frac{F_z^{(n)}}{\left(\frac{dF_z}{d\Delta z}\right)^{(n)}} \quad (A21)$$

$$\frac{dF_z}{d\Delta z} = 1 + \frac{I_T (2a_y + 3b_y \bar{y})}{\lambda \bar{y}^3 (a_y + b_y \bar{y})^2} \frac{d\bar{y}}{d\Delta z} \quad (A22)$$

Equation A7 was used to obtain:

$$\frac{d\bar{y}}{d\Delta z} = \frac{1}{2} \frac{dy_2}{d\Delta z} = \frac{\left(\frac{g}{g_c J} - \frac{q_r}{\dot{W}_{st}}\right)}{2 (H_{v_2} + y_2 f_1 v_{G_2}^2)} \quad (A23)$$

The substitution of equations A22 and A23 into equation A21 provides a procedure to obtain a Δz to satisfy equation A20.

The equation for the vapor pressure of steam is given in Keenan and Keyes (8) as:

$$P = \phi(T_s) = P_c e^{-2.303 u_E}$$

$$\phi'(T_s) = 2.303 \phi(T_s) u'_E$$

where

$$u_E = \frac{x}{T_{sk}} \left[\frac{a_p + b_p x + c_p x^3 + e_p x^4}{(1 + d_p x)} \right]$$

$$u'_E = - \frac{du_E}{DT_{sk}} = \frac{du_E^\dagger}{dx}$$

$$= \left[\frac{a_p + u_E(1 - T_c d_p) + 2x(b_p + u_E d_p) + 4C_p x^3 + 5e_p x^4}{(T_c - x)(1 + d_p x)} \right]$$

$$\begin{aligned} x &= T_c - T_{sk} \\ T_c &= 647.27^\circ\text{K} \\ a_p &= 3.3463130 \\ b_p &= 4.14113 \times 10^{-2} \\ C_p &= 7.515484 \times 10^{-9} \\ d_p &= 1.3794481 \times 10^{-2} \\ e_p &= 6.56444 \times 10^{-11} \\ P_c &= \text{critical pressure of the steam (psia)} \end{aligned}$$

The determination of v_G is straight forward. Equation 13 page 15 in Keenan and Keyes(8), was applied with the use of a conversion factor ($\frac{1}{62.43}$) to change the units to ft^3/lbm .

I_T was subsequently evaluated numerically using a nine point Gauss-Quadrature integration formula.

[†] I_T was evaluated in degrees Kelvin, since the equation for v_G and P were given in degrees Kelvin.

I-B. THE WELLBORE

(Calculation of Casing Temperature and U_o)

1) Single Casing

At steady state, with no heat source or sink, the heat transferred through every resistance in the wellbore must be equal(6, 12). Thus referring to Figure 1 obtain:

$$\begin{aligned}
 \delta q &= 2\pi r_1 dz h_s (T_s - T_1) \\
 &= 2\pi dz k_1 \frac{T_1 - T_2}{\ln \frac{r_2}{r_1}} \\
 &= 2\pi r_2 dz f_{12} (T_2 + T_3) (T_2^2 + T_3^2) (T_2 - T_3) \\
 &\quad + 2\pi dz k_c \frac{(T_2 - T_3)}{\ln \frac{r_3}{r_2}} \\
 &= 2\pi dz k_2 \frac{T_3 - T_4}{\ln \frac{r_4}{r_3}}
 \end{aligned}$$

Solving for the temperature differences, adding them and subsequently solving for δq , yields:

$$\delta q = 2\pi r_4 dz U_o (T_s - T_4) = \frac{2\pi dz (T_s - T_4)}{\frac{1}{r_1 h_s} + \frac{\ln \frac{r_2}{r_1}}{k_1} + \frac{1}{r_2 h_r + \frac{k_c}{\ln \frac{r_3}{r_2}}} + \frac{\ln \frac{r_4}{r_3}}{k_2}} \quad (B1)$$

where

$$h_r = f_{12} \sigma (T_2 + T_3) (T_2^2 + T_3^2) \quad (B2)$$

$$f_{12} = \frac{1}{\frac{1}{\epsilon_1} + \frac{r_2}{r_3} \left(\frac{1}{\epsilon_2} - 1 \right)}$$

Solving equation B1 for U_o obtain:

$$\frac{1}{U_o} = \frac{r_4}{r_1 h_s} + \frac{r_4 \ln \frac{r_2}{r_1}}{k_1} + \frac{r_4}{r_2 h_r + \frac{k_c}{\ln \frac{r_3}{r_2}}} + \frac{r_4 \ln \frac{r_4}{r_3}}{k_2} \quad (B3)$$

Letting:

$$C_3 = \frac{r_4}{r_1 h_s} + \frac{r_4 \ln \frac{r_2}{r_1}}{k_1} + \frac{r_4 \ln \frac{r_4}{r_3}}{k_2}$$

$$C_4 = \frac{r_2}{r_4} f_{12} \sigma$$

$$C_5 = \frac{1}{r_4 \ln \frac{r_3}{r_2}}$$

Equation B3 may be rewritten as:

$$U_o = \frac{1}{C_3} \left[1 - \frac{1}{C_3 C_4 (T_2 + T_3) (T_2^2 + T_3^2) + C_3 C_5 k_c + 1} \right] \quad (B4)$$

Where k_c is defined as the effective thermal conductivity of the air in the annular space. It combines natural convection and conduction. From Groeber(6) page 317 obtain:

$$k_c = 0.11k_a (Pr \cdot Gr)^{0.29} \quad (2000^+ < Pr \cdot Gr < 10^6) \quad (B5)$$

$$k_c = k_a \quad (2000 > Pr \cdot Gr)$$

$$Pr \cdot Gr = \frac{7500 (12)^3 C_{Pa} (T_2 - T_3) P_a^2 M^2 (r_3 - r_2)^3 g}{\mu_a k_a T_M^3 R^2}$$

$$= \frac{C_6 C_{Pa} (T_2 - T_3)}{\mu_a k_a T_M^3} \quad (B6)$$

where

$$T_M = \frac{T_2 + T_3}{2}$$

$$C_6 = \frac{7500 (12)^3 P_a^2 M^2 (r_3 - r_2)^3 g}{R^2}$$

$$\mu_a = F(T_M)$$

$$C_{Pa} = F(T_M) \quad \text{all functions are assumed linear}$$

$$k_a = F(T_M)$$

Since the heat transferred through the wellbore must equal the heat entering the formation obtain:

+ the relationship is extrapolated from the specified minimum of 6000 to 2000. The error introduced is negligible.

$$2\pi r_4 (T_s - T_c) U_o dz = \frac{2\pi k_E}{f(t)} (T_c - T_E) dz$$

Solving for T_c yields:

$$T_c = T_s - \frac{k_E (T_s - T_E)}{k_E + U_o r_4 f(t)}$$

or

$$F_T = T_c - T_s + \frac{k_E (T_s - T_E)}{k_E + U_o r_4 f(t)} = 0 \quad (B7)$$

Assuming the temperature drops across the tubing and the casing are fairly constant, may write:

$$T_2 = T_s - \Delta_t$$

$$T_3 = T_c + \Delta_c$$

Thus, T_c remains the only variable in equation B4. Substitution of equation B4 into equation B7 results in one equation in T_c . Due to the complicated nature of the equation, a trial and error solution appears to be necessary. For a fast and reasonably reliable convergence procedure, Newton's method was employed.

Thus:

$$T_c^{(n+1)} = T_c^{(n)} - \frac{F_T^{(n)}}{\left(\frac{F_T}{dT_c}\right)^{(n)}}$$

$$\frac{dF_T}{dT_C} = \frac{dF_T}{dT_3} = 1 - \frac{k_E (T_S - T_E) r_4 f(t)}{(k_E + U_O r_4 f(t))^2} \frac{dU_O}{dT_3}$$

The differentiation of equation B3 yields:

$$\frac{dU_O}{dT_3} = \frac{C_4 (T_2^2 + 2T_2 T_3 + 3T_3^2) + C_5 k'_C}{[C_3 \cdot C_4 (T_2 + T_3) (T_2^2 + T_3^2) + C_3 \cdot C_5 k_C + 1]^2}$$

where

$$k'_C = \frac{dk_C}{dT_3} = k_C \left[\frac{k'_a}{k_a} + \frac{0.29}{Pr \cdot Gr} \frac{d(Pr \cdot Gr)}{dT_3} \right] \quad (B8)$$

$$\begin{aligned} \frac{d(Pr \cdot Gr)}{dT_3} = \frac{C_6}{2k_a \mu_a T_M^3} \{ & 2C'_{P_a} (T_2 - T_3) - 2C_{P_a} \\ & - (T_2 - T_3) C_{P_a} \left[\frac{3}{T_M} + \frac{2k'_a}{k_a} + \frac{2\mu'_a}{\mu_a} \right] \} \end{aligned} \quad (B9)$$

$$C'_{P_a} = \frac{dC_{P_a}}{dT_3} = \frac{1}{2} \frac{dC_{P_a}}{dT_M}$$

$$k'_a = \frac{dk_a}{dT_3} = \frac{1}{2} \frac{dk_a}{dT_M}$$

$$\mu'_a = \frac{d\mu_a}{dT_3} = \frac{1}{2} \frac{d\mu_a}{dT_M}$$

After T_c has been calculated by the prescribed or other iterative procedure, Δ_t and Δ_c may be calculated and T_c redetermined. Comparison of the two values so obtained will indicate whether or not another iteration is required.

$$\Delta_t = T_s - T_2 = \frac{k_E (T_c - T_E)}{f(t)} \left(\frac{1}{r_1 h_s} + \frac{\ln \frac{r_2}{r_1}}{k_1} \right)$$

$$\Delta_c = T_3 - T_c = \frac{k_E (T_c - T_E)}{f(t)} \frac{\ln \frac{r_4}{r_3}}{k_2}$$

2) Double Casing

The treatment of the double casing situation is similar to the single casing case. Thus, emphasis will be placed on the differences in the two problems. Following the same procedure as in Part (1) obtain:

$$\begin{aligned} \frac{1}{U_o} = & \frac{r_6}{r_1 h_s} + \frac{r_6 \ln \frac{r_2}{r_1}}{k_1} + \frac{1}{\frac{r_2}{r_6} h_r + \frac{k_c}{r_6 \ln \frac{r_3}{r_2}}} \\ & + \frac{r_6 \ln \frac{r_4}{r_3}}{k_2} + \frac{1}{\frac{r_4}{r_6} h_{r_2} + \frac{k_{c2}}{r_6 \ln \frac{r_5}{r_4}}} + \frac{r_6 \ln \frac{r_6}{r_5}}{k_3} \end{aligned} \quad (B10)$$

$$\begin{aligned} \frac{1}{U_{OM}} = & \frac{r_4}{r_1 h_s} + \frac{r_4 \ln \frac{r_2}{r_1}}{k_1} + \frac{1}{\frac{r_2}{r_4} h_r + \frac{k_c}{r_4 \ln \frac{r_3}{r_2}}} \\ & + \frac{r_4 \ln \frac{r_4}{r_3}}{k_2} \end{aligned} \quad (B11)$$

where U_{OM} is defined as the overall heat transfer coefficient between the first casing and the steam, based on the outside area of the first casing.

Equation B11 may be rewritten employing the definitions of Part (1).

$$U_{OM} = \frac{1}{C_3} \left[1 - \frac{1}{C_3 \cdot C_4 (T_2 + T_3) (T_2^2 + T_3^2) + C_3 \cdot C_5 \cdot k_c + 1} \right] \quad (B12)$$

Similarly:

$$U_o = \frac{1}{C_{10}} \left[1 - \frac{F1 + F2}{C_{10} \cdot F1 \cdot F2 + F1 + F2} \right] \quad (B13)$$

where

$$C_{10} = \frac{r_6}{r_1 h_s} + \frac{r_6 \ln \frac{r_2}{r_1}}{k_1} + \frac{r_6 \ln \frac{r_4}{r_3}}{k_2} + \frac{r_6 \ln \frac{r_6}{r_5}}{k_3}$$

$$F1 = C_{11} (T_2 + T_3) (T_2^2 + T_3^2) + C_{12} k_c$$

$$F2 = C_{13} (T_4 + T_5) (T_4^2 + T_5^2) + C_{14} k_{c2}$$

$$C_{11} = \frac{r_2}{r_6} f_{12} \sigma$$

$$C_{12} = \frac{1}{r_6 \ln \frac{r_3}{r_2}}$$

$$C_{13} = \frac{r_4}{r_6} f_{34} \sigma$$

$$C_{14} = \frac{1}{r_6 \ln \frac{r_5}{r_4}}$$

$$f_{34} = \frac{1}{\frac{1}{\epsilon_3} + \frac{r_4}{r_5} \left(\frac{1}{\epsilon_4} - 1 \right)}$$

k_c may be evaluated by equation B5, while k_{c_2} may be calculated similarly using the properties of the second annulus

$$(T_{M_2} = \frac{T_4 + T_5}{2}, \text{ etc.}).$$

At steady state, the heat conducted through any part of the wellbore must be equal to the heat absorbed by the formation. Thus:

$$\frac{2\pi k_E (T_c - T_E) dz}{f(t)} = 2\pi r_6 U_O (T_s - T_c) dz \quad (B14)$$

$$\frac{2\pi k_E (T_c - T_E) dz}{f(t)} = 2\pi r_4 U_{OM} (T_s - T_4) dz \quad (B15)$$

Equation B15 may be rearranged to yield:

$$T_C = T_E + \frac{r_4 f(t) U_{OM} (T_S - T_4)}{k_E} \quad (B16)$$

The solution of equation B14 for T_C yields:

$$T_C = T_S + \frac{k_E (T_E - T_S)}{k_E + r_6 f(t) U_O}$$

or

$$F_{T2} = T_C - T_S + \frac{k_E (T_S - T_E)}{k_E + r_6 f(t) U_O} = 0 \quad (B17)$$

It is now possible to substitute equations B12, B13 and B16 into equation B17 and derive one implicit equation in T_4 (or T_3). Newton's method was used as a basis for convergence.

$$T_3^{(n+1)} = T_3^{(n)} - \frac{F_{T2}^{(n)}}{\left(\frac{dF_{T2}}{dT_3}\right)^{(n)}} \quad (B18)$$

Assuming

$$T_2 = T_S - \Delta_t$$

$$T_4 = T_3 - \Delta_c$$

$$T_5 = T_C + \Delta_{c2}$$

$$\frac{dF_{T2}}{dT_3} = \frac{dT_C}{dT_3} - \frac{k_E (T_S - T_E) r_6 f(t)}{(k_E + r_6 f(t) U_O)^2} \frac{dU_O}{dT_3}$$

The differentiation of equation B16 yields:

$$\frac{dT_c}{dT_3} = \frac{r_4 f(t)}{k_E} \left[-U_{OM} + (T_s - T_4) \frac{dU_{OM}}{dT_3} \right]$$

The differentiation of equation B12 yields

$$\frac{dU_{OM}}{dT_3} = \frac{C_4(T_2^2 + 2T_2 T_3 + 3T_3^2) + C_5 k'_C}{[(C_3 \cdot C_4 (T_2 + T_3) (T_2^2 + T_3^2) + C_3 \cdot C_5 k_C + 1)^2]}$$

where k'_C is defined by equation B8.

The differentiation of equation B13 yields:

$$\frac{dU_o}{dT_3} = \frac{1}{C_{10}} \left[\frac{(F1 + F2) [C_{10} (F1 \cdot F2' + F2 \cdot F1') + F1' + F2']}{[C_{10} \cdot F1 \cdot F2 + F1 + F2]^2} - \frac{F1' + F2'}{(C_{10} \cdot F1 \cdot F2 + F1 + F2)} \right]$$

where

$$F1' = \frac{dF1}{dT_3} = C_{11}(T_2^2 + 2T_2 T_3 + 3T_3^2) + C_{12}k'_C$$

$$F2' = \frac{dF2}{dT_3} = C_{13}(T_5^2 + 2T_4 T_5 + 3T_4^2) + C_{14}k'_C$$

$$+ [C_{13}(T_4^2 + 2T_4 T_5 + 3T_5^2) + C_{14}\bar{k}'_C] \frac{dT_c}{dT_3}$$

$$k'_C = \frac{\partial k_{C2}}{\partial T_3} \quad (T_5 \text{ is constant})$$

$$\bar{k}'_C = \frac{\partial k_{C2}}{\partial T_5} \quad (T_3 \text{ is constant})$$

After a value for T_c has been obtained, better values for Δ_t , Δ_c and Δ_{c_2} may be calculated.

$$\Delta_t = T_s - T_2 = \frac{k_E (T_c - T_E)}{f(t)} \left(\frac{1}{r_1 h_s} + \frac{\ln \frac{r_2}{r_1}}{k_1} \right)$$

$$\Delta_c = T_3 - T_4 = \frac{k_E (T_c - T_E)}{f(t)} \frac{\ln \frac{r_4}{r_3}}{k_2}$$

$$\Delta_{c_2} = T_5 - T_c = \frac{k_E (T_c - T_E)}{f(t)} \frac{\ln \frac{r_6}{r_5}}{k_3}$$

It was found that Δ_t , Δ_c and Δ_{c_2} were quite small, and did not affect the value for T_c . Therefore, no iterations were necessary to correct the values for Δ_t , Δ_c and Δ_{c_2} .

I-C. THE FORMATION

(Calculation of $f(t)$)

The partial differential equation describing conduction in cylindrical coordinate for a constant thermal conductivity is(12):

$$\frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t} \quad (C1)$$

Assuming: radial symmetry

no axial (vertical) conduction

Equation C1 reduces to:

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} = \frac{1}{\alpha} \frac{\partial T}{\partial t} \quad (C2)$$

Defining:

$$T^* = T - T_s \quad \text{Therefore, } T = T^* + T_s$$

$$T_o^* = T_E - T_s$$

The substitution of the expression for T into equation C2 yields:

$$\frac{\partial^2 T^*}{\partial r^2} + \frac{1}{r} \frac{\partial T^*}{\partial r} = \frac{1}{\alpha} \left(\frac{\partial T^*}{\partial t} + \frac{\partial T_s}{\partial t} \right) \quad (C3)$$

Since $\frac{\partial T_s}{\partial t}$ is generally quite small, it was assumed to be zero, whereby equation C3 reduces to

$$\frac{\partial^2 T^*}{\partial r^2} + \frac{1}{r} \frac{\partial T^*}{\partial r} = \frac{1}{\alpha} \frac{\partial T^*}{\partial t} \quad (C4)$$

Equation C4 is solved in Carslaw and Jaeger(4) page 337 employing Laplace transforms and a contour integral. The solution is subject to the following initial and boundary conditions:

Initial condition:

at $t = 0$ and $r \geq r_c$, $T^* = T_o^*$ (i.e. $T = T_E$)

Boundary conditions:

1) Temperature (T) is finite at an infinite radius

2) Heat conducted through the wellbore is equal to the heat entering the formation. In mathematical form obtain:

$$2\pi r_c U_o (T_s - T_c) dz = - k_E 2\pi r_c \left. \frac{\partial T}{\partial r} \right|_{r=r_c} dz$$

or

$$U_o (T_c - T_s) = k_E \left. \frac{\partial T}{\partial r} \right|_{r=r_c}$$

Thus:

$$\left. \frac{\partial T^*}{\partial r} \right|_{r=r_c} = \frac{U_o}{k_E} T^* \left|_{r=r_c} \right. \quad (C5)$$

as

$$\frac{\partial T^*}{\partial r} = \frac{\partial T}{\partial r} \quad \text{and} \quad T_c - T_s = T^* \left|_{r=r_c} \right.$$

The assumption of a constant heat transfer coefficient and the stated boundary and initial conditions lead to the following solution of equation C4.

$$T^*(r,t) = \frac{-2 \frac{U_o}{k_E} T_o^*}{\pi} \int_0^\infty e^{-\alpha u^2 t} du$$

$$\cdot \left[\frac{J_o(ur) \{uY_1(ur_c) + \frac{U_o}{k_E} Y_o(ur_c)\} - Y_o(ur) \{uJ_1(ur_c) + \frac{U_o}{k_E} J_o(ur_c)\}}{\{uJ_1(ur_c) + \frac{U_o}{k_E} J_o(ur_c)\}^2 + \{uY_1(ur_c) + \frac{U_o}{k_E} Y_o(ur_c)\}^2} \right] \frac{du}{u} \quad (C6)$$

where $J_o(ur)$, $J_1(ur)$, $Y_o(ur)$ and $Y_1(ur)$ are all Bessel functions. Since the heat flux at the wellbore-formation interface is the desired property, may write:

$$\frac{\partial T}{\partial r} \Big|_{r=r_c} = \frac{\partial T^*}{\partial r} \Big|_{r=r_c} = \frac{U_o}{k_E} T^* \Big|_{r=r_c} \quad (\text{from equation C5})$$

Thus:

$$\frac{\partial T}{\partial r} \Big|_{r=r_c} = \frac{-2 \left(\frac{U_o}{k_E}\right)^2 T_o^*}{\pi} \int_0^\infty e^{-\alpha u^2 t} du$$

$$\cdot \left[\frac{J_o(ur_c) uY_1(ur_c) - Y_o(ur_c) uJ_1(ur_c)}{\{uJ_1(ur_c) + \frac{U_o}{k_E} J_o(ur_c)\}^2 + \{uY_1(ur_c) + \frac{U_o}{k_E} Y_o(ur_c)\}^2} \right] \frac{du}{u}$$

But

$$-J_o(ur_c) Y_1(ur_c) + Y_o(ur_c) J_1(ur_c) = \frac{2}{\pi(ur_c)}$$

Thus :

$$\left. \frac{\partial T}{\partial r} \right|_{r=r_c} = \frac{4U_T^2 T_o^*}{\pi^2 r_c} \int_0^\infty \left[\frac{e^{-t_D \omega^2}}{\{\omega J_1(\omega) + U_T J_0(\omega)\}^2 + \{\omega Y_1(\omega) + U_T Y_0(\omega)\}^2} \right] \frac{d\omega}{\omega} \quad (C7)$$

where :

$$\omega = u r_c$$

$$U_T = \frac{U_o r_c}{k_E} \quad (C8)$$

$$t_D = \frac{\alpha t}{r_c^2} \quad (C9)$$

From the definition of $f(t)$ obtain:

$$2\pi r_c k_E dz \left. \frac{\partial T}{\partial r} \right|_{r=r_c} = - \frac{2\pi k_E (T_c - T_E)}{f(t)} dz$$

Solving for $f(t)$ yields

$$f(t) = \frac{T_E - T_c}{r_c \left. \frac{\partial T}{\partial r} \right|_{r=r_c}}$$

From equation C5 obtain

$$T_c = T_s + \frac{r_c}{U_T} \left. \frac{\partial T}{\partial r} \right|_{r=r_c}$$

Thus

$$f(t) = \frac{T_E - T_s}{r_c \left. \frac{\partial T}{\partial r} \right|_{r=r_c}} - \frac{1}{U_T} \quad (C10)$$

The substitution of equation C10 into equation C7 yields:

$$f(t) = \frac{1}{I_t} \left(\frac{\pi}{2U_T} \right)^2 - \frac{1}{U_T} \quad (C11)$$

where:

$$I_t = \int_0^{\infty} \left[\frac{e^{-t_D \omega^2}}{\{\omega J_1(\omega) + U_T J_0(\omega)\}^2 + \{\omega Y_1(\omega) + U_T Y_0(\omega)\}^2} \right] \frac{d\omega}{\omega} \quad (C12)$$

$$= \int_0^{\infty} f(\omega) d\omega$$

Direct numerical evaluation of I_t is difficult, as $\lim_{\omega \rightarrow 0} f(\omega) = \infty$

To circumvent this difficulty, the Integral was evaluated in two parts.

for $\omega < 10^{-3}$, the following situation arises:

$$\omega J_1(\omega) \rightarrow 0$$

$$U_T J_0(\omega) \rightarrow U_T$$

$$\omega Y_1(\omega) \rightarrow -0.6366$$

$$U_T Y_0(\omega) \rightarrow \frac{2U_T}{\pi} \ln(\omega)$$

$$\frac{e^{-t_D \omega^2}}{\omega} \rightarrow 1.0$$

Thus

$$f(\omega) \rightarrow \frac{1}{\omega(U_T^2 + (\frac{2U_T}{\pi} \ln \omega - 0.6366)^2)}$$

$$I_{t_1} = \int_0^{\omega_1} f(\omega) d\omega = \int_{\omega=0}^{\omega=\omega_1} \frac{\pi}{2U_T^2} \frac{d\bar{\omega}}{(1 + \bar{\omega}^2)} \quad (C13)$$

where

$$\bar{\omega} = \frac{2}{\pi} \ln \omega - \frac{0.6366}{U_T}$$

If $\omega_1 < 10^{-5}$, $\bar{\omega}$ (for $U_T = \infty$) < -7.34

Thus:

$$\bar{\omega}^2 + 1 \approx \bar{\omega}^2$$

If ω_1 is taken as 10^{-5} , the error introduced by the approximation is believed to be negligible. With the approximation, equation C13 may be modified and integrated.

$$\begin{aligned} I_{t_1} &= - \frac{\pi}{2U_T^2} \left[\frac{1}{\bar{\omega}} \right]_{\omega=0}^{\omega=10^{-5}} = - \frac{\pi}{2U_T^2} \left[\frac{1}{\frac{2}{\pi} \ln \omega - \frac{0.6366}{U_T}} \right]_{\omega=0}^{\omega=10^{-5}} \\ &= - \frac{\pi}{2U_T^2} \left[\frac{1}{\frac{2}{\pi} \ln (10^{-5}) - \frac{0.6366}{U_T}} \right] \end{aligned} \quad (C14)$$

and

$$I_{t_2} = \int_{10^{-5}}^{\infty} \left[\frac{e^{-t_D}}{\{\omega J_1(\omega) + U_T J_0(\omega)\}^2 + \{\omega Y_1(\omega) + U_T Y_0(\omega)\}^2} \right] \frac{d\omega}{\omega} \quad (C15)$$

Following the substitution of polynomial approximations for the various Bessel functions (page 369 in Ref. 1), the integral

I_{t_2} was evaluated numerically employing a nine point Gauss Quadrature integration formula. In an attempt to optimize the numerical integration, the interval size ($\Delta\omega$) was increased as ω increased.

The values for $f(t)$ calculated, were found to be comparable to those presented graphically by Ramey(11).

A P P E N D I X II
THE COMPUTER PROGRAM

A computer program was written to solve the four equations developed in Appendix I simultaneously. Upon specification of time and steam temperature; depth, quality, casing temperature and $f(t)$ are determined by trial and error to within specified errors. The temperature decrement is then adjusted linearly to yield an increment in depth approximately equal to the specified interval. An iterative procedure based on successive linear adjustments of the temperature decrement is applied to obtain values for the bottom of the well. Any numerical integration occurring in the solution was performed using a nine point Gauss Quadrature formula. Only the negative sample points and their corresponding weighting factors are read in, the other values are generated by the computer with a mirroring procedure. This mirroring routine requires the total number of points to be odd. To guard against endless loops resulting from non-convergence, both subroutines calculating the overall heat transfer coefficient contain error-exits.

A separate program was employed to fit the tabulated data for the heat of vaporization and C_{sL} to polynomials in temperature. A seventh order polynomial yielded a maximum deviation of 0.1% between the calculated and tabulated heat of vaporization. As the effect of C_{sL} on the overall solution is minor, a fourth order polynomial fit resulted in sufficient accuracy. The computer program is limited by the following

restrictions:

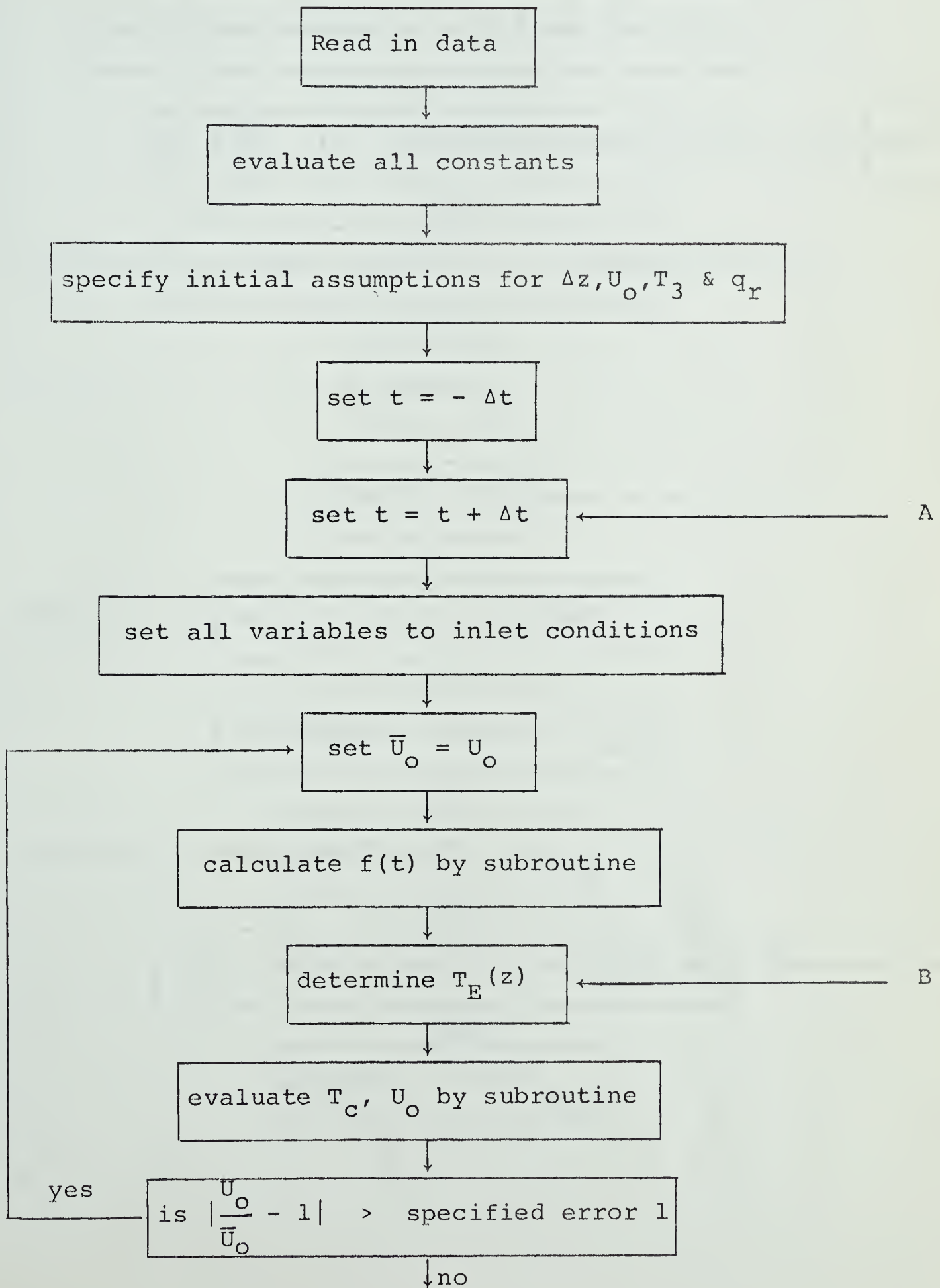
$$1) \quad 50^{\circ}\text{C} < T_s < 374^{\circ}\text{C}$$

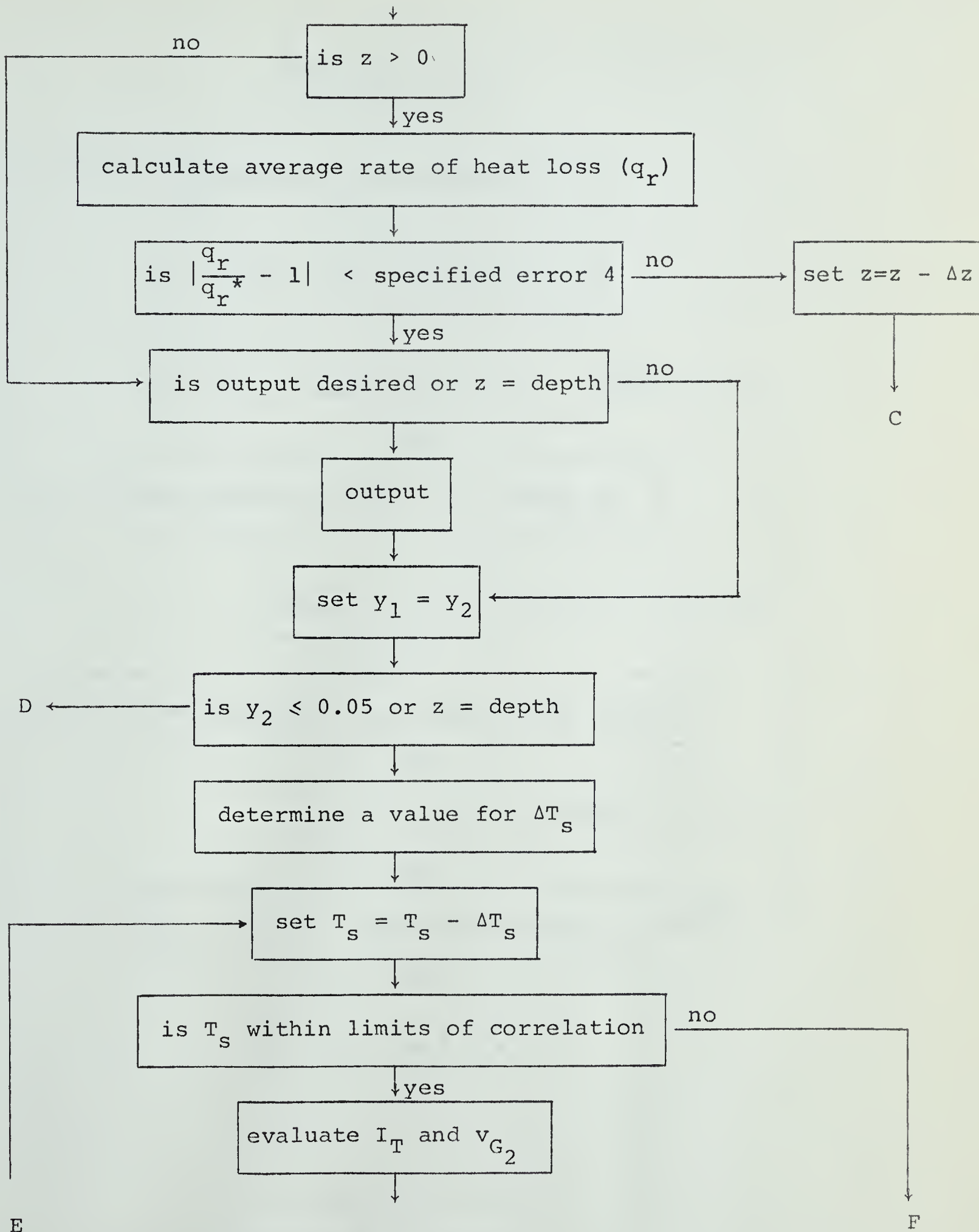
$$2) \quad \text{Pr} \cdot \text{Gr} < 10^6$$

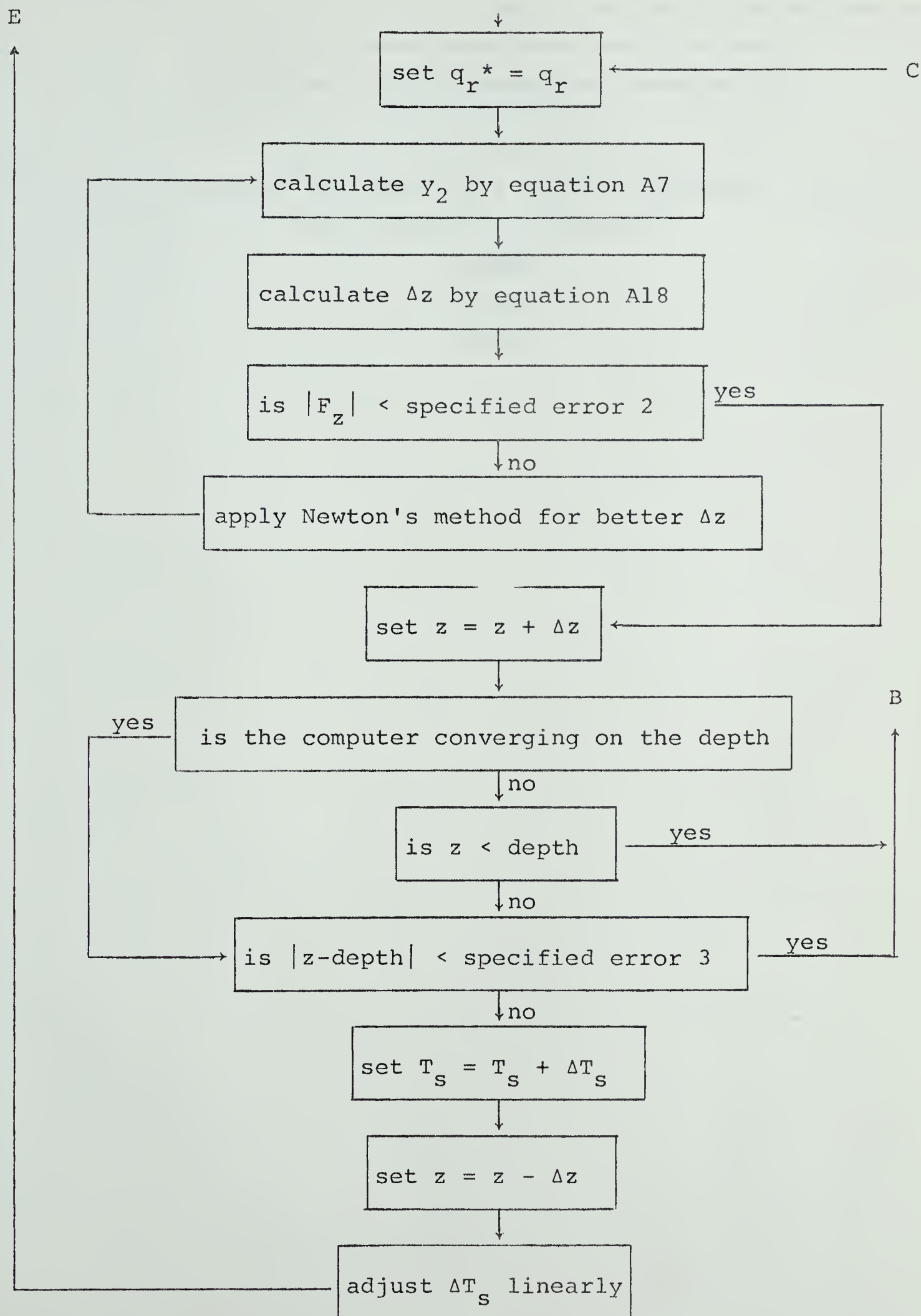
$$3) \quad 0.05 \leq y$$

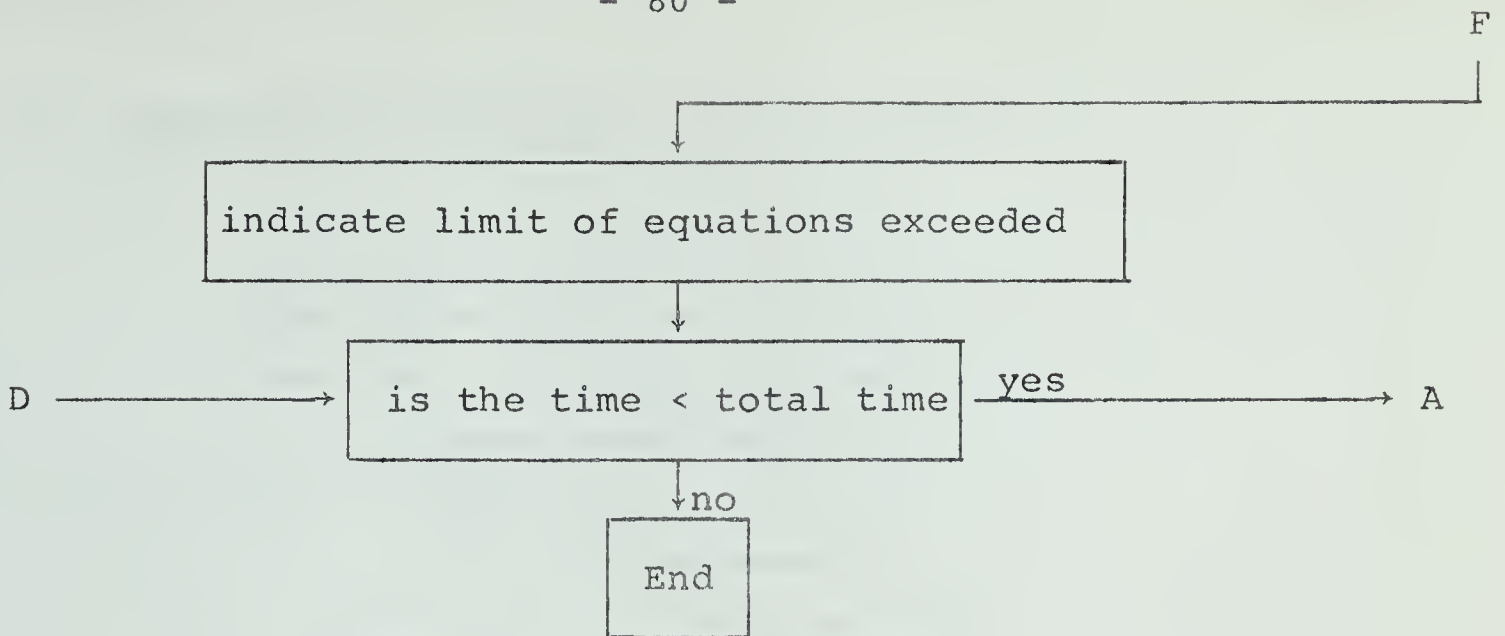
II-A. FLOW DIAGRAMS

i) Main Program

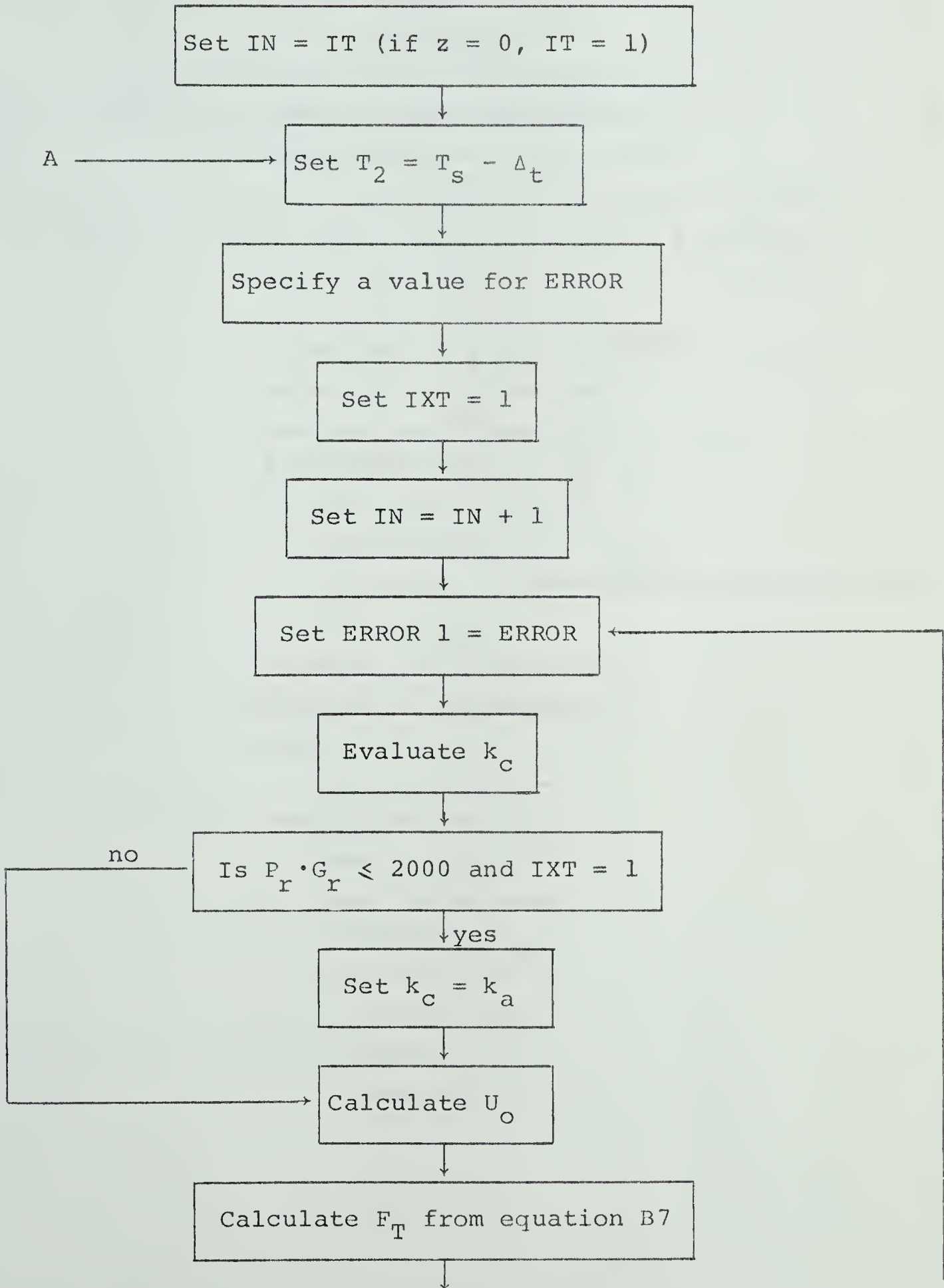


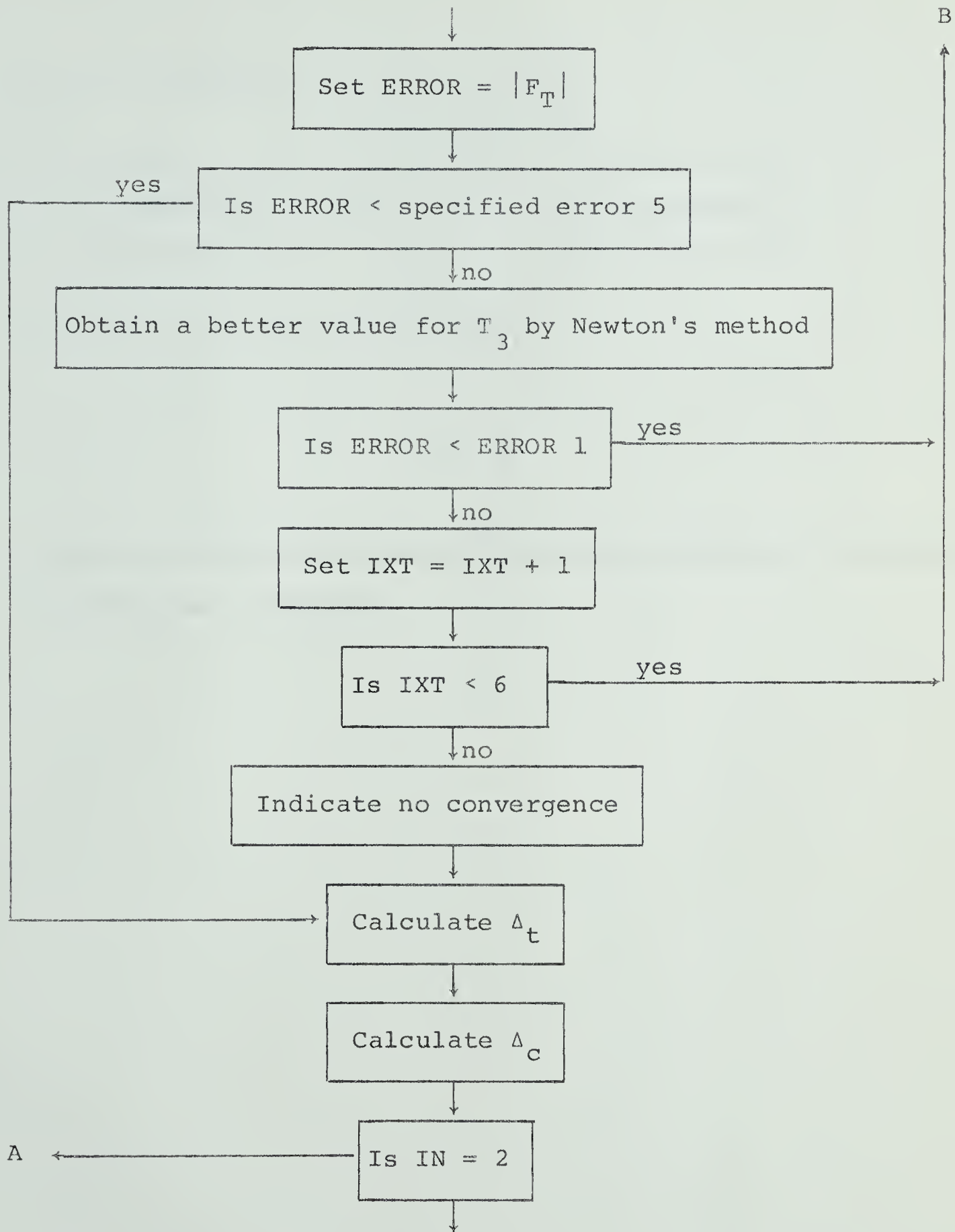


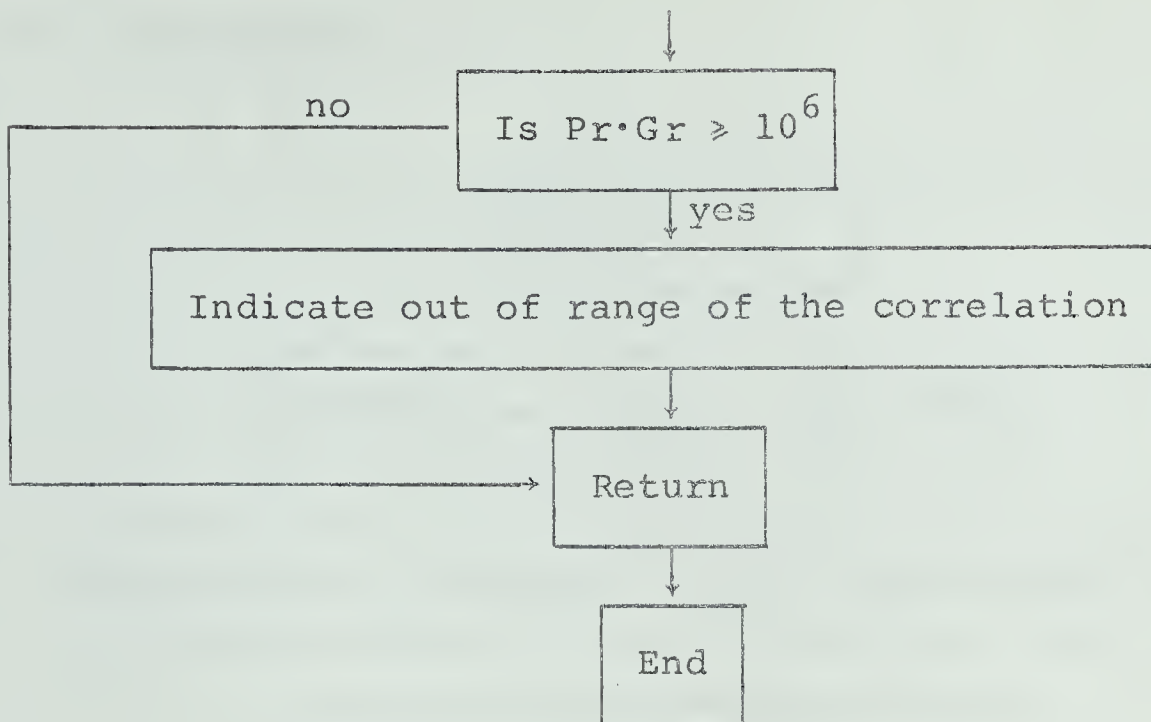




ii) Subroutine Coefficient (single casing)

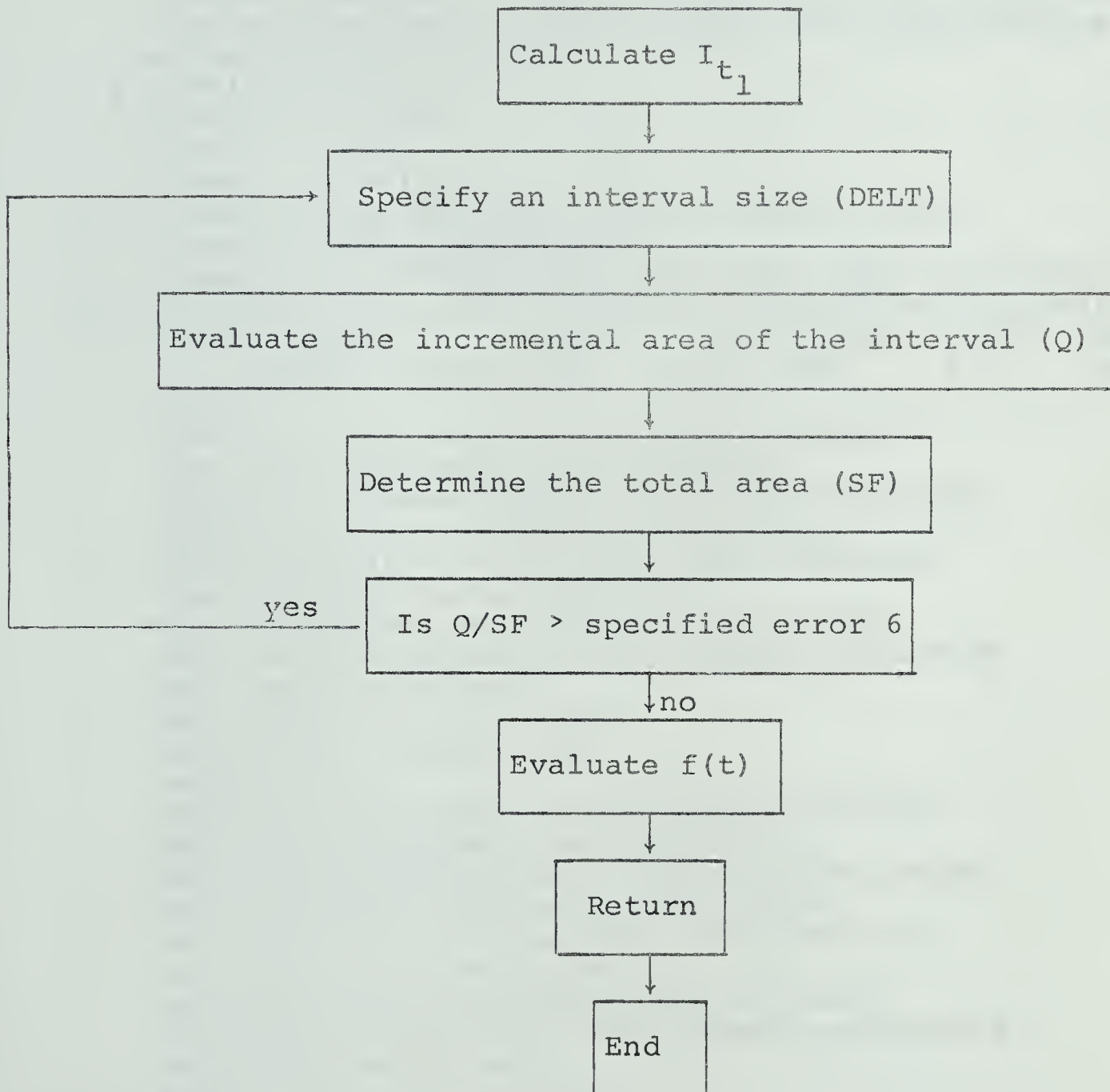






The flow diagram for the double casing situation is similar and thus not included.

iii) Subroutine FTIME



II-B FORTRAN SOURCE LISTS

1) MAIN PROGRAM

```

$IBFTC ASTRI  NODECK
COMMON PI,UK,DX,TR,TIMEF,LY,V(20),H(20),ERT1,IT,R4,R6,TC,UO,ST,
1AVISC1,AVISC2,AK1,AK2,ACP1,ACP2,ECON,ET,ERC1,ERC2,A,B,C,D,A1,A2,
2A3,AAA,BB,CC,E,F,FF,T3
1 FORMAT(1H 6F13.6)
2 FORMAT(1H 6E12.3)
3 FORMAT(1H 3I3)
5 FORMAT (1H 5F14.8)
100 FORMAT(1HL16H WELLBORE AFTER ,F10.2,5H DAYS)
101 FORMAT(1HL11H INPUT DATA/)
102 FORMAT(1H 33HTEMPERATURE LESS THAN LOWER LIMIT,3E14.6)
103 FORMAT(1H F12.2,2F13.4,E14.4,F10.4,2F13.4,3F11.4,I5)
110 FORMAT(1HK122H          DEPTH          STEAM TEMP    HEAT LOSS    CUM HEAT LO
1SS  QUALITY  CASING TEMP    EARTH TEMP      F(T)      OV. COEFF.    F.
2LOSS/)
READ (5,1)ACP1,ACP2,AK1,AK2,AVISC1,AVISC2
READ (5,1)QA,QB,QUAL,WST,STEMP1,PA
READ(5,1) ECON,EDENS,ECP,ETEMP1,ETEMP2,HEVAP
READ(5,1)EM1,EM2,EM3,EM4,DELT,DEPTH
READ (5,1)FILMC,DTFI,COND1,COND2,COND3,DX
READ (5,1)R1,R2,R3,R4,R5,R6
READ (5,2)ERM1,ERM2,ERM3,ERM4,ERC1,ERT1
READ (5,2)TOTIME,ADIST,TMULT,TMULT2,VGI,HRLIQ
READ (5,3)LM,NC,IWM
READ (5,5) (V(L),L=1,LM)
READ (5,5) (H(L),L=1,LM)
WRITE(6,101)
WRITE(6,1)ACP1,ACP2,AK1,AK2,AVISC1,AVISC2
WRITE(6,1)QA,QB,QUAL,WST,STEMP1,PA
WRITE(6,1)ECON,EDENS,ECP,ETEMP1,ETEMP2,HEVAP
WRITE(6,1)EM1,EM2,EM3,EM4,DELT,DEPTH
WRITE(6,1)FILMC,DTFI,COND1,COND2,COND3,DX
WRITE(6,1)R1,R2,R3,R4,R5,R6
WRITE(6,2)ERM1,ERM2,ERM3,ERM4,ERC1,ERT1
WRITE(6,2)TOTIME,ADIST,TMULT,TMULT2,VGI,HRLIQ
WRITE(6,3)LM,NC,IWM
WRITE (6,5) (V(L),L=1,LM)
WRITE (6,5)(H(L),L=1,LM)
PI=3.1415927
LY=2*LM-1
LT=LY+1
LS=LM+1
DO 9 M=LS,LY
LTM=LT-M
V(M)=-V(LTM)
9 H(M)=H(LTM)
V(LT)=0.
H(LT)=0.
C  CONSTANTS FOR HEAT TRANSFER COEFFICIENT
HTLOSS=400.0
UO=1.0

```


FORTRAN SOURCE LIST ASTRI

```

T3=(STEMP1+ETEMP1)/2.
A1=12./FILMC/R1+ALOG(R2/R1)/COND1
A2=ALOG(R4/R3)/COND2
A=R4*(A1+A2)/12.
B=R2*0.1713/(1./EM1+R2*(1./EM2-1.)/R3)/R4*0.1E-07
C=12./(R4*ALOG(R3/R2))
D=(R3-R2)*((R3-R2)*28.9*PA/10.73)**2*32.2*0.75E+04
CTIME=144.*ECON/(EDENS*ECP*R4**2)*24.
CUO=R4/ECON/12.
IF(NC.EQ.1)GO TO 10
A3=ALOG(R6/R5)/COND3
AAA=R6*(A1+A2+A3)/12.
BB=R4*B/R6
CC=R4*C/R6
E=12./R6/ALOG(R5/R4)
F=R4*0.1713/(1./EM3+R4*(1./EM4-1.)/R5)/R6*0.1E-07
FF=(R5-R4)*((R5-R4)*28.9*PA/10.73)**2*32.2*0.75E+04
CTIME=CTIME*(R4/R6)**2
CUO=R6/ECON/12.
C  CONSTANTS FOR FRICTION LOSS
10 AA=3.346313
   AB=0.0414113
   AC=0.751548E-08
   AD=0.013794481
   AE=0.656444E-10
   AT=647.27
   Y=WST*6./R1/PI
   CZ1=(WST/(62.43*25.*PI*R1**2))**2/(64.4*777.97)
   CZ3=-0.2E-07/(5.04*1.61*6.243*1.47*R1**3)*Y**2
C  ST=STEAM TEMPERATURE
C  ZTP=DISTANCE DOWN THE WELLBORE
C  QUAA=AVERAGE QUALITY
C  UO=OVERALL HEAT TRANSFER COEFFICIENT
C  TIMEF=F(T)
   HEATIN=HRLIQ+QUAL*HEVAP
   TIME=DELT/TMULT-DELT*TMULT2
11 TIME=TIME*TMULT+DELT*TMULT2
   ZTP=0.
   DZTP=ADIST
   ST=STEMP1
   CHLOSS=0.
   HXLOSS=0.
   IXT=1
   DTF=DTFI
   IT=1
   INTC=0
   QUA2=QUAL
   V2S=(VGI*62.43)**2
   STF=STEMP1-460.
   ENTH2=HEVAP
   IW=IWM
   WRITE(6,100)TIME

```


FORTTRAN SOURCE LIST ASTRI

```

WRITE(6,110)
TR=TIME*CTIME
12 UK=UO*CUO
ERR=UO
CALL FTIME
13 ET=ETEMP1+ZTP*ETEMP2
CALL COEFF (IXT)
IF (ABS(ERR/UO-1.)*GT.ERM1)GO TO 12
IF (IT.LT.2)GO TO 21
17 HTLOSS=PI*ECON*((TC1-ET1)+(TC-ET))/TIMEF
IF (ABS(HLOSS/HTLOSS-1.)*LT.ERM4)GO TO 22
ZTP=ZTP-DZTP
GO TO 16
22 CHLOSS=CHLOSS+HTLOSS*DZTP
HXLOSS=HTLOSS
21 ET1=ET
IT=IT+1
TC1=TC
VIS=V2S
IF ((IW.LT.IWM).AND.(INTC.LE.1))GO TO 30
ETF=ET-460.
TCF=TC-460.
FLOSS=CHLOSS/WST/HEATIN
WRITE(6,103)ZTP,STF,HXLOSS,CHLOSS,QUA2,TCF,ETF,TIMEF,UO,FLOSS,IXT
IW=0
30 QUA1=QUA2
IW=IW+1
IF (QUA2.LE.0.05)GO TO 25
IF (INTC.GE.2)GO TO 25
DTF=DTF*ADIST/DZTP
14 ST=ST-DTF
DTK=5.*DTF/9.
ENTH1=ENTH2
STF=ST-460.
CPL=2.8317766-0.020296439*STF+0.81715712E-04*STF**2-
10.14154051E-06*STF**3+0.92129424E-10*STF**4
ENTH2=0.15914927E+04-0.10402037E+02*STF+0.81499083E-01*STF**2-
10.36702958E-03*STF**3+0.96928323E-06*STF**4-0.15126743E-08*STF**5
2+0.12907291E-11*STF**6-0.46713135E-15*STF**7
SFXP=0.
X1=647.27-ST*5./9.
X2=X1+DTK
IF (X2.GT.590.)GO TO 24
DO 15 I=1,LT
XV=(X1*(1.-V(I))+X2*(1.+V(I)))/2.
IF (I.EQ.LT)XV=X2
UE=XV*(AA+AB*XV+AC*XV**3+AE*XV**4)/((1.+AD*XV)*(AT-XV))
DUE=(AA+UE*(1.-AT*AD)+2.*(AB+UE*AD)*XV+4.*AC*XV**3+5.*AE*XV**4)/
1((AT-XV)*(1.+AD*XV))
P=218.167*EXP(-UE*2.303)
TAU=1./(AT-XV)
GIT=82.546*TAU-0.16246E+06*TAU**2

```


FORTRAN SOURCE LIST ASTRI

```
G2T=0.21828-0.12697E+06*TAU**2
G3T=0.3635E-03-0.6768E-07*(1000.*TAU)**24
BO=1.89-2641.62*TAU*10.***(80870.*TAU**2)
B1=BO*(1.+BO*TAU*P*G1T+(BO*TAU*P)**3*G2T-(BO*TAU*P)**12*G3T)
VG=4.55504/(P*TAU)+B1
15 SFXP=SFXP-H(I)*2.303*P*DUE/VG
V2S=VG**2
16 HLOSS=HTLOSS
18 QUA2=(QUA1*(ENTH1+QUA1*CZ1*V1S)+CPL*DTF+(1./777.97-HLOSS/WST)*DZTP
1)/(ENTH2+QUA2*CZ1*V2S)
QUAA=(QUA1+QUA2)/2.
SZP=DZTP-SFXP/(CZ3*QUAA**2*(QA+QB*QUAA))*DTK
IF(ABS(SZP).LT.ERM2)GO TO 19
DER=1.0-SFXP/CZ3*(2.*QA+3.*QB*QUAA)/(QUAA**3*(QA+QB*QUAA)**2)*
1(HLOSS/WST-1./777.97)/(2.*(ENTH2+QUA2*CZ1*V2S))*DTK
DZTP=DZTP-SZP/DER
GO TO 18
19 ZTP=ZTP+DZTP
IF(INTC.EQ.2)GO TO 20
IF(ZTP.LT.DEPTH)GO TO 13
20 DELZTP=ZTP-DEPTH
INTC=2
IF(ABS(DELZTP).LE.ERM3)GO TO 13
ST=ST+DTF
ENTH2=ENTH1
ZTP=ZTP-DZTP
DTF=(1.-DELZTP/DZTP)*DTF
GO TO 14
24 WRITE(6,102)ST,ZTP,X2
25 IF(TIME.LT.TOTIME)GO TO 11
CALL EXIT
END
```


ii) SUBROUTINE FOR THE HEAT TRANSFER COEFFICIENT (1 CASING)

```
$IBFTC COEFF1
  SUBROUTINE COEFF (IXT)
    COMMON PI,UK,DX,TR,TIMEF,LY,V(20),H(20),ERT1,IT,R4,R6,TC,UO,ST,
    1AVISC1,AVISC2,AK1,AK2,ACP1,ACP2,ECON,ET,ERC1,ERC2,A,B,C,D,A1,A2,
    2A3,AAA,BB,CC,E,F,FF,T3
  10 FORMAT(1HK40H LIMIT OF K CORRELATION EXCEEDED IN C-1 ,E14.6,2F10.5
    1/)
  11 FORMAT(1HK23H NO CONVERGENCE IN C-1 ,5E14.5/)
    IN=IT
    G=R4*TIMEF/12.
  1 T2=ST-DIFF1
    ERROR=1.0E+20
    IXT=1
    IN=IN+1
  2 ERROR1=ERROR
    TM=(T2+T3)/2.
    DEL=T2-T3
    TMF=TM-460.
    AVISC=AVISC1+AVISC2*TMF
    AK=AK1+AK2*TMF
    CP=ACP1+ACP2*TMF
    DEN3=AK*AVISC*TM**3
    PRGR=D*DEL*CP/DEN3
    PAK=0.11*AK*PRGR**0.29
    IF((PRGR.LE.2000.).AND.(IXT.EQ.1))PAK=AK
  3 DEN=A*B*(T2+T3)*(T2**2+T3**2)+A*C*PAK+1.
    UO=(1.-1./DEN)/A
    T4=T3-DIFF2
    FT=T4-ST+ECON*(ST-ET)/(UO*G+ECON)
    ERROR=ABS(FT)
    IF(ERROR.LT.ERC1)GO TO 4
    DPRGR=D*(DEL*ACP2-2.*CP-DEL*CP*(3./TM+AK2/AK+AVISC2/AVISC))/
    1(DEN3*2.)
    DPAK=PAK*(AK2/AK/2.+0.29*DPRGR/PRGR)
    IF((PRGR.LE.2000.).AND.(IXT.EQ.1))DPAK=AK2/2.
    DEFT=1.-G*ECON*(ST-ET)*(B*(T2**2+2.*T2*T3+3.*T3**2)+C*DPAK)/(DEN*
    1(UO*G+ECON))**2
    T3=T3-FT/DEFT
    IF(ERROR.LT.ERROR1)GO TO 2
    IXT=IXT+1
    IF(IXT.LE.6)GO TO 2
    WRITE(6,11)ERROR,ERROR1,PRGR,PAK,AK
  4 DELQ=ECON*(T4-ET)/TIMEF
    DIFF1=DELQ*A1
    DIFF2=DELQ*A2
    IF(IN.EQ.2)GO TO 1
  6 TC=T4
    IF(PRGR.GE.0.1E+07)WRITE(6,10)PRGR,PAK,AK
    RETURN
  END
```


iii) SUBROUTINE FOR THE HEAT TRANSFER COEFFICIENT (2 CASINGS)

\$IBFTC COEFF2

SUBROUTINE COEFF (IXT)

COMMON PI,UK,DX,TR,TIMEF,LY,V(20),H(20),FRT1,IT,R4,R6,TC,UO,ST,
1 AVISC1,AVISC2,AK1,AK2,ACP1,ACP2,ECON,ET,ERC1,ERC2,A,B,C,D,A1,A2,
2 A3,AAA,BB,CC,E,F,FF,T3

10 FORMAT(1HK40H LIMIT OF K CORRELATION EXCEEDED IN C-2 ,2E14.6,
14F10.5/)

11 FORMAT(1HK23H NO CONVERGENCE IN C-2 ,9E14.5/)

IN=IT

G=R4*TIMEF/12.

GG=R6*TIMEF/12.

1 T2=ST-DIFF1

ERROR=1.0E+20

IXT=1

IN=IN+1

2 ERROR1=ERROR

TM=(T2+T3)/2.

DEL=T2-T3

TMF=TM-460.0

AVISC=AVISC1+AVISC2*TMF

AK=AK1+AK2*TMF

CP=ACP1+ACP2*TMF

DEN3=AK*AVISC*TM**3

PRGR=D*DEL*CP/DEN3

PAK=0.11*AK*PRGR**0.29

IF((PRGR.LE.2000.).AND.(IXT.EQ.1))PAK=AK

3 FF1= B*(T2+T3)*(T2**2+T3**2)

DEN1= A*(FF1+ C*PAK)+1.

UUO=(1.-1./DEN1)/A

T4=T3-DIFF2

T6=UUO*(ST-T4)*G/ECON+ET

T5=T6+DIFF3

DELA=ABS(T4-T5)

TMA=(T4+T5)/2.

TMF=TMA-460.0

AVISCA=AVISC1+AVISC2*TMF

AKA=AK1+AK2*TMF

CPA=ACP1+ACP2*TMF

DEN4=AKA*AVISCA*TMA**3

PRGR2=FF*DELA*CPA/DEN4

PAK2=0.11*AKA*PRGR2**0.29

IF((PRGR2.LE.2000.).AND.(IXT.EQ.1))PAK2=AKA

F1=BB*FF1/B+CC*PAK

F2=F*(T4+T5)*(T4**2+T5**2)+E*PAK2

DEN2=AAA*F1*F2+F1+F2

UO=(1.-(F1+F2)/DEN2)/AAA

FT=T6-ST+ECON*(ST-ET)/(UO*GG+ECON)

ERROR=ABS(FT)

IF(ERROR.LT.ERC1)GO TO 4

DPRGR=D*(DEL*ACP2-2.*CP-DEL*CP*(3./TM+AK2/AK+AVISC2/AVISC))/

1(DEN3*2.)

DPAK=PAK*(AK2/AK/2.+0.29*DPRGR/PRGR)

IF((PRGR.LE.2000.).AND.(IXT.EQ.1))DPAK=AK2/2.

FORTRAN SOURCE LIST COEFF2

```
IF((PRGR2.LE.2000.).AND.(IXT.EQ.1))GO TO 5
DPRGR2=FF*(DELA*ACP2-2.*CPA-DELA*CPA*(3./TMA+AK2/AKA+AVISC2/AVISCA
1)))/(2.*DEN4)
DPAK2=PAK2*(AK2/AKA/2.+0.29*DPRGR2/PRGR2)
DPRGR3=DPRGR2+FF*2.*CPA/DEN4
DPAK3=PAK2*(AK2/AKA/2.+0.29*DPRGR3/PRGR2)
GO TO 6
5 DPAK2=AK2/2.
DPAK3=AK2/2.
6 FFP=B*(T2**2+2.*T2*T3+3.*T3**2)
FFP1=FFP+C*DPAK
FP1=FFP*BB/B+CC*DPAK
DT6=G*(FFP1*(ST-T4)/DEN1**2-UUO)/ECON
FP2=F*(3.*T4**2+2.*T4*T5+T5**2)+E*DPAK3+(F*(T4**2+2.*T4*T5+
13.*T5**2)+E*DPAK2)*DT6
DUOT=((F1+F2)*(AAA*(FP2*F1+FP1*F2)+FP1+FP2)/DEN2**2-(FP1+FP2)/DEN2
1)/AAA
DEFT=DT6-ECON*GG*(ST-ET)/(UO*GG+ECON)**2*DUOT
T3=T3-FT/DEFT
IF(ERROR.LT.ERROR1)GO TO 2
IXT=IXT+1
IF(IXT.LE.6)GO TO 2
WRITE(6,11)ERROR,ERROR1,PRGR,PAK,AK,PRGR2,PAK2,AKA
4 DELQ=ECON*(T6-ET)/TIMEF
DIFF1=DELQ*A1
DIFF2=DELQ*A2
DIFF3=DELQ*A3
IF(IN.EQ.2)GO TO 1
TC=T6
IF((PRGR.GT.1.0E+06).OR.(PRGR2.GT.1.0E+06))WRITE(6,10)PRGR,PRGR2,
1AK,PAK,AKA,PAK2
RETURN
END
```


iv) SUBROUTINE FOR THE TIME FUNCTION

\$IBFTC TIME

```

SUBROUTINE FTIME
COMMON PI,UK,DX,TR,TIMEF,LY,V(20),H(20),ERT1,IT,R4,R6,TC,UO,ST,
1AVISC1,AVISC2,AK1,AK2,ACP1,ACP2,ECON,ET,ERC1,ERC2,A,B,C,D,A1,A2,
2A3,AAA,BB,CC,E,F,FF,T3
SF=-PI/(2.*UK**2*(2./PI*ALOG(0.1F-04)-0.6366/UK))
Z2=0.1E-04
DELT=DX/TR
1 Z1=Z2
Q=0.
IF(DELT.GT.0.001)GO TO 2
DELT=10.*DELT
GO TO 3
2 DELT=1.2*DELT
3 Z2=Z2+DELT
CON=0.
DO 6 L=1,LY
X=(Z1*(1.-V(L))+Z2*(1.+V(L)))/2.
IF(X.GT.3.)GO TO 4
U=X/3.
F1=1.0-2.2499997*U**2
F2=X*(0.5-0.56249985*U**2)
F3=2./PI*ALOG(X/2.)*F1+0.36746691+0.60559366*U**2
F4=2./PI*X*ALOG(X/2.)*F2-0.6366198+0.2212091*U**2
IF(U.LT.0.1E-02)GO TO 5
F1=F1+1.2656208*U**4-0.3163866*U**6+0.0444479*U**8
F2=F2+(0.21093573-0.03954289*U**2+0.00443319*U**4)*X*U**4
F3=F3-0.74350384*U**4+0.25300117*U**6-0.04261214*U**8
F4=F4+(2.1682709-1.3164827*U**2+0.3123951*U**4)*U**4
IF(X.LT.0.1) GO TO 5
F1=F1-(0.3944E-02-0.21E-03*U**2)*U**10
F2=F2-(0.31761E-03-0.1109E-04*U**2)*X*U**10
F3=F3+(0.427916E-02-0.24846E-03*U**2)*U**10
F4=F4-(0.400976E-01-0.27873E-02*U**2)*U**10
GO TO 5
4 U=3./X
FB=0.79788456-0.00000077*U-0.00552740*U**2-0.00009512*U**3+
10.00137237*U**4-0.00072805*U**5+0.00014476*U**6
THETA=X-0.78539816-0.04166397*U-0.00003954*U**2+0.00262573*U**3
1-0.00054125*U**4-0.00029333*U**5+0.00013558*U**6
FBB=FB/X**0.5
F1=FBB*COS(THETA)
F3=FBB*SIN(THETA)
FB=0.79788456+0.00000156*U+0.01659667*U**2+0.00017105*U**3-
10.00249511*U**4+0.00113653*U**5-0.00020033*U**6
THETA=X-2.35619449+0.12499612*U+0.00005650*U**2-0.00637879*U**3
1+0.00074348*U**4+0.00079824*U**5-0.00029166*U**6
FBB=FB/X**0.5
F2=FBB*COS(THETA)
F4=FBB*X*SIN(THETA)
5 DEN=X*((UK*F1+X*F2)**2+(UK*F3+F4)**2)
XP=-TR*X**2
IF(ABS(XP).GT.40.) GO TO 6

```


FORTRAN SOURCE LIST TIME

```
Q=Q+DELT*H(L)*EXP(XP)/(2.*DEN)
6 CONTINUE
SF=SF+Q
IF((Q/SF.GT.ERT1).OR.(X.LT.2.))GO TO 1
TIMEF=(PI/(2.*UK))**2/SF-1./UK
RETURN
END
```


II-C. NOMENCLATURE OF COMPUTER INPUT

ACP1	Specific heat of the air in the annulus at 0°F ($\text{Btu}/\text{lb}_m^{\circ}\text{F}$)
ACP2	Specific heat change with temperature ($\text{Btu}/\text{lb}_m^{\circ}\text{F}^2$)
AK1	Thermal conductivity of the air at 0°F ($\text{Btu}/\text{hr}.\text{ft}^{\circ}\text{F}$)
AK2	Thermal conductivity change with temperature ($\text{Btu}/\text{hr}.\text{ft}^{\circ}\text{F}^2$)
AVISCL	Viscosity of the air at 0°F ($\text{lb}_m/\text{hr}.\text{ft}.$)
AVISCL2	Viscosity change with temperature ($\text{lb}_m/\text{hr}.\text{ft}^{\circ}\text{F}$)
QA	Coefficients for friction factor vs quality equation $f_G = \text{QA} + \text{QB} \cdot \text{quality}$
QB	
QUAL	Quality of the inlet steam (fraction)
WST	Mass flowrate of the steam ($\text{lb}_m/\text{hr}.$)
STEMP1	Temperature of the inlet steam ($^{\circ}\text{R}$)
PA	Pressure of the air in the annulus (psia)
ECON	Earth conductivity ($\text{Btu}/\text{hr}.\text{ft}^{\circ}\text{F}$)
EDENS	Earth density (lb_m/ft^3)
ECP	Earth specific heat ($\text{Btu}/\text{lb}_m^{\circ}\text{F}$)
ETEMP1	Earth surface temperature ($^{\circ}\text{R}$)
ETEMP2	Geothermal gradient ($^{\circ}\text{R}/\text{ft}$)
HEVAP	Heat of vaporization at inlet temperature (Btu/lb_m)
EM1	ϵ_1 = emissivity of outside of the tubing
EM2	ϵ_2 = emissivity of inside of the casing
EM3	ϵ_3 = emissivity of outside of the casing
EM4	ϵ_4 = emissivity of inside of the second casing

DELT	Basic time increment (days)
DEPTH	Total depth of the well (ft)
FILMC	Film coefficient for the inside of the tubing (Btu/hr.ft. ² °F)
DTFI	Initial steam temperature decrement (°F)
COND1	Thermal conductivity of the tubing (Btu/hr.ft.°F)
COND2	Thermal conductivity of the casing (Btu/hr.ft.°F)
COND3	Thermal conductivity of the second casing (Btu/hr.ft.°F)
DX	Initial interval for evaluating I_{t_2}
R1	Inside radius of the tubing (in)
R2	Outside radius of the tubing (in)
R3	Inside radius of the casing (in)
R4	Outside radius of the casing (in)
R5	Inside radius of the second casing (in)
R6	Outside radius of the second casing (in)
ERM1	Specified error 1 (see main flow diagram)
ERM2	Specified error 2 (see main flow diagram)
ERM3	Specified error 3 (see main flow diagram)
ERM4	Specified error 4 (see main flow diagram)
ERC1	Specified error 5 (see subroutine coefficient flow diagram)
ERT1	Specified error 6 (see subroutine FTIME flow diagram)
TOTIME	Maximum injection time to be considered (days)
ADIST	Approximate depth interval to be considered (ft)

TMULT	coefficient to obtain $t = t*TMULT + DELT*TMULT2$
TMULT2	coefficient to obtain $t = t*TMULT + DELT*TMULT2$
VGI	Specific volume of water vapor at inlet temperature (ft ³ /lb _m)
HRLIQ	Enthalpy of liquid at inlet temperature - enthalpy of liquid at earth temperature (Btu/lb _m)
LM	Number of sample points for Gauss Quadrature (neg.) (must be odd)
NC	Number of casings (1 or 2)
IWM	Number of points calculated before one is printed out
V _i	Sample points for Gauss Quadrature (neg. only)
H _i	Weighting factors for Gauss Quadrature for above sample points

A P P E N D I X I I I

SAMPLE CALCULATIONS

The results of one computer run were obtained as B.C.D. cards, spaced and printed on the line printer, and subsequently included in this section. As B.C.D. card output is restricted to 80 print positions, compared to the line printer's 132; $f(t)$, average rate of heat loss, earth temperature, U_o and IXT, which were included in the written output, were not punched out. All the input data is presented as a print out of the actual computer input, thus sections II-B and II-C are required to correlate the number to the variable and its units. This section also contains some plots of friction factor vs quality and the resulting relationships. An attempt to fit the points of Figure 15 to a polynomial yielded little success, attributable to the odd shape of the curve. As a result straight line approximations were made for all cases, rather than dividing the curve into ranges.

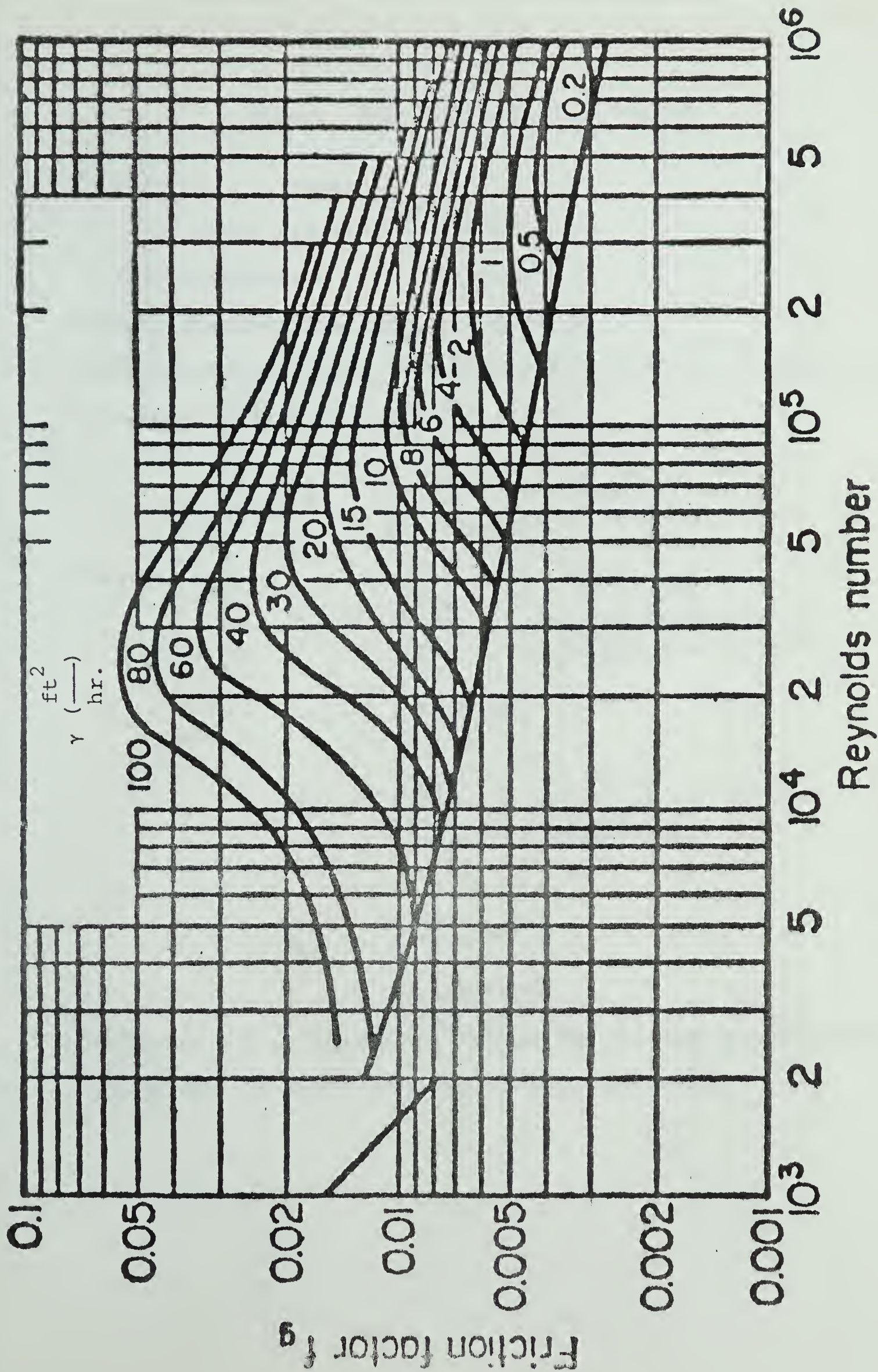


FIG. 14. TWO PHASE FRICTION FACTOR VERSUS VAPOR PHASE REYNOLDS NUMBER

III-A. TWO PHASE FRICTION FACTOR

Figure 15 Data:

W_{st} = steam flowrate = 3000 lb_m/hr

Injection pressure = 500 psia

Tubing diameter (inside) = 2.441 in

From Keenan and Keyes(8) obtain the values for saturated vapor viscosity and liquid density.

$$\mu_G = 5.45 \times 10^{-2} \text{ lb}_m/\text{hr-ft}$$

$$\rho_L = 50.7 \text{ lb}_m/\text{ft}^3$$

Thus:

$$\begin{aligned} Y_a &= \frac{2W_{st}}{\pi r_1^2 \mu_G} = \frac{2 \times 3000}{\pi \left(\frac{2.441}{2 \times 12} \right)^2 5.45 \times 10^{-2}} \\ &= 3.45 \times 10^5 \end{aligned}$$

$$Y_b = \frac{W_{st}}{2\pi r_1^2 \rho_L} = \frac{3000}{\pi \left(\frac{2.441}{12} \right)^2 50.7} = 92.5$$

and

$$R_{EG} = 3.45 \times 10^5 y$$

$$\gamma = 92.5 (1 - y)$$

With the use of Figure 14, values for the two phase friction factor may be determined for various qualities

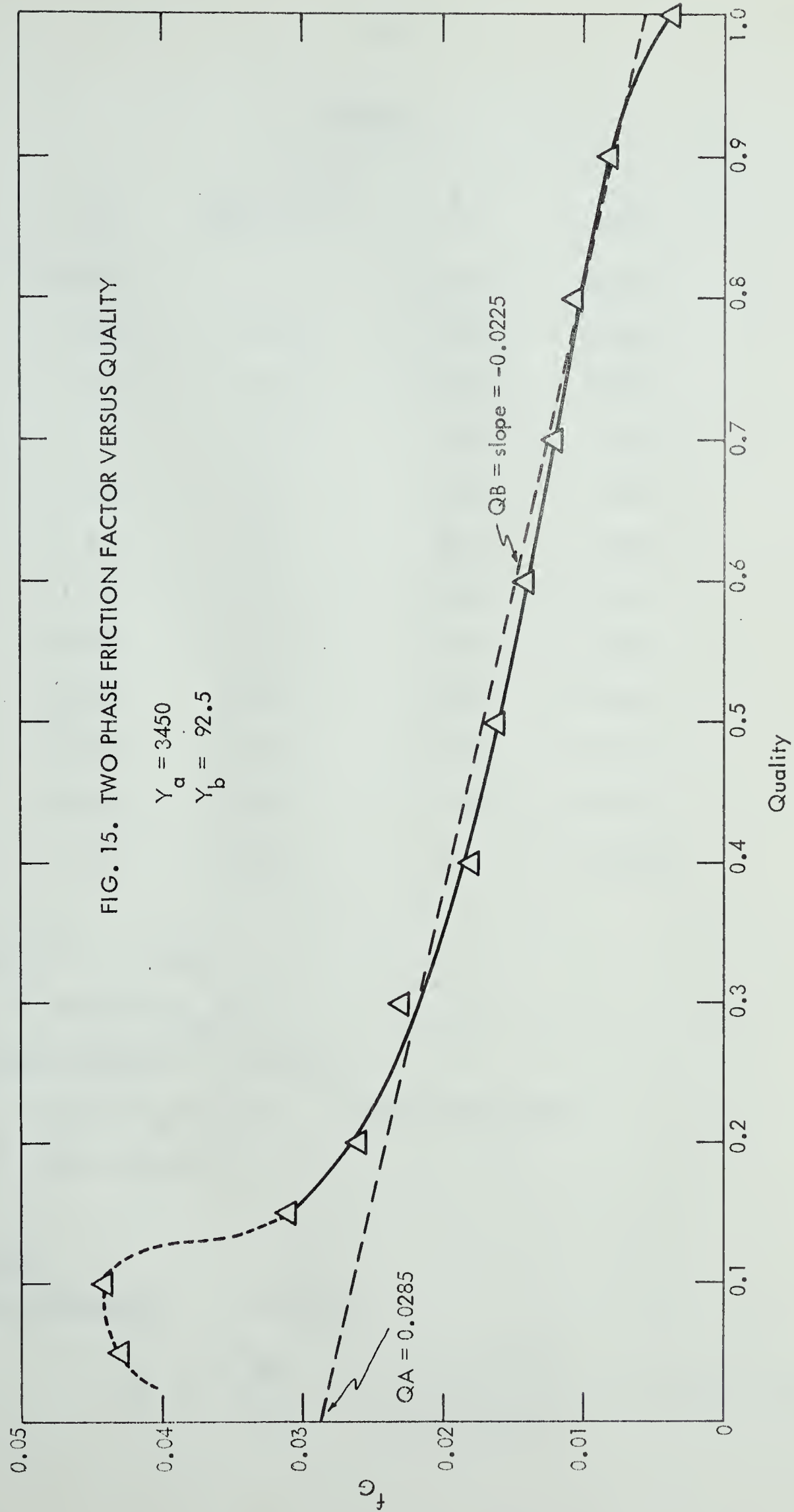


Table I

<u>y</u>	<u>R_{EG} x 10⁻⁴</u>	<u>γ</u>	<u>f_G</u>
0.05	1.73	87.8	0.043
0.10	3.45	83.3	0.044
0.15	5.16	78.6	0.031
0.20	6.90	74.0	0.026
0.30	10.4	64.8	0.023
0.40	13.8	55.5	0.018
0.50	17.3	46.3	0.016
0.60	20.7	37.0	0.014
0.70	24.2	27.8	0.012
0.80	27.6	18.5	0.010
0.90	31.1	9.25	0.008
1.00	34.5	0	0.0035

Figure 17 Data:

$$W_{st} = 10,500 \text{ lb}_m/\text{hr}$$

Average pressure \approx 700 psia

$$\mu_G = 0.08 \text{ lb}_m/\text{hr.ft.} \quad (\text{from Perry(10)})$$

$$\rho_L = 48.8 \text{ lb}_m/\text{ft}^3$$

Part i)

Tubing diameter = 1.995 in

$$Y_a = \frac{2W_{st}}{\pi r_l \mu_G} = \frac{2 \times 10,500}{\pi \left(\frac{1.995}{2 \times 12}\right) 0.08} = 1.005 \times 10^6$$

$$Y_b = \frac{W_{st}}{2\pi r_1 \rho_L} = \frac{10,500}{\pi \left(\frac{1.995}{12}\right) 48.8} = 411$$

Thus for Part i) have:

$$\begin{aligned} R_{EG} &= 1.005 \times 10^6 y \\ \gamma &= 411 (1 - y) \end{aligned}$$

Part ii)

Tubing diameter = 2.441 in

$$\begin{aligned} R_{EG} &= 8.21 \times 10^5 y \\ \gamma &= 337 (1 - y) \end{aligned}$$

Part iii)

Tubing diameter = 2.992 in

$$\begin{aligned} R_{EG} &= 6.69 \times 10^5 y \\ \gamma &= 274 (1 - y) \end{aligned}$$

The γ values involved are out of the range of Figure 14, thus Figure 16 was obtained and the curves extrapolated. The values so obtained were plotted on Figure 14 and a curve was then sketched in.

FIG. 16. TWO PHASE FRICTION FACTOR VERSUS χ

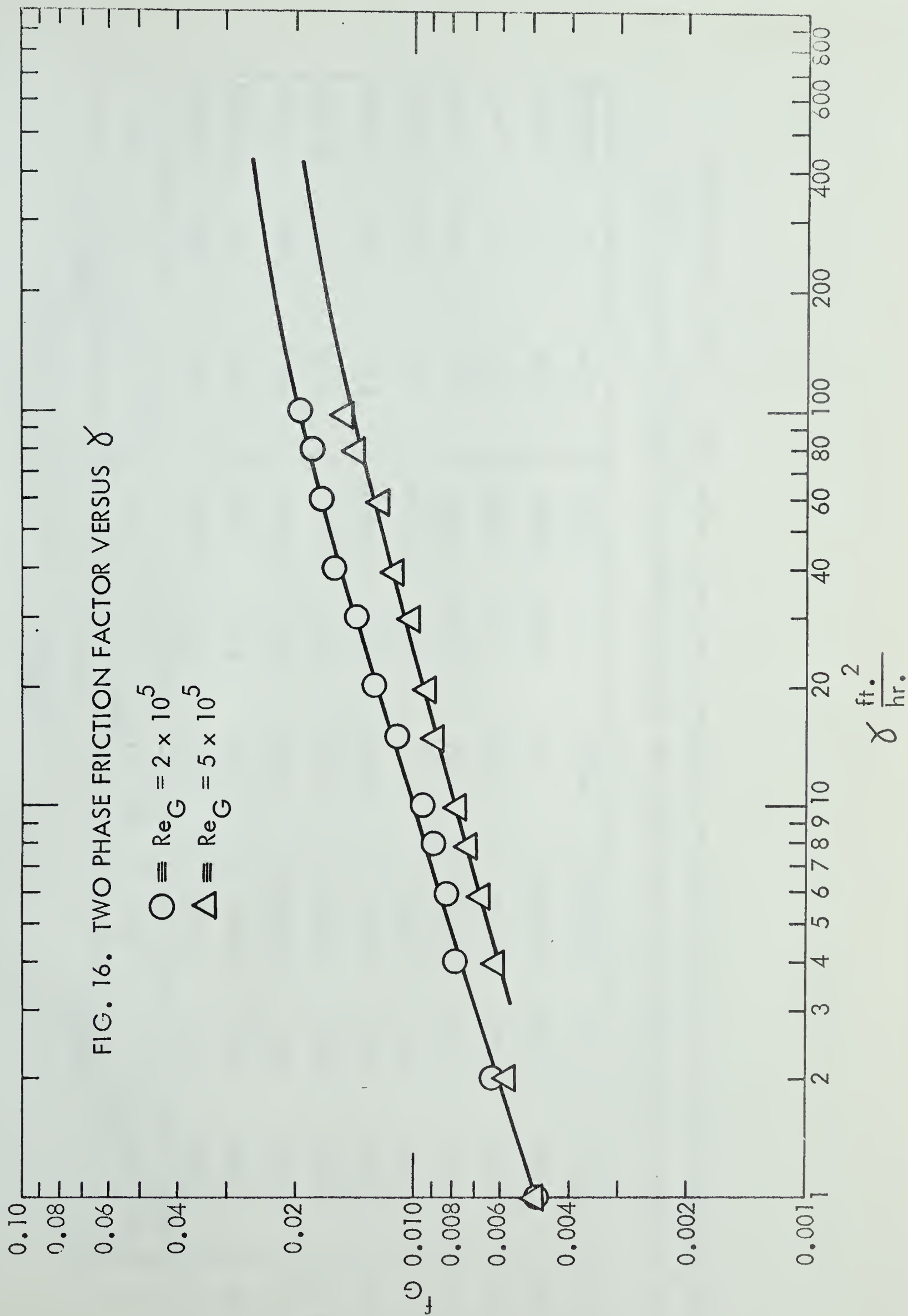
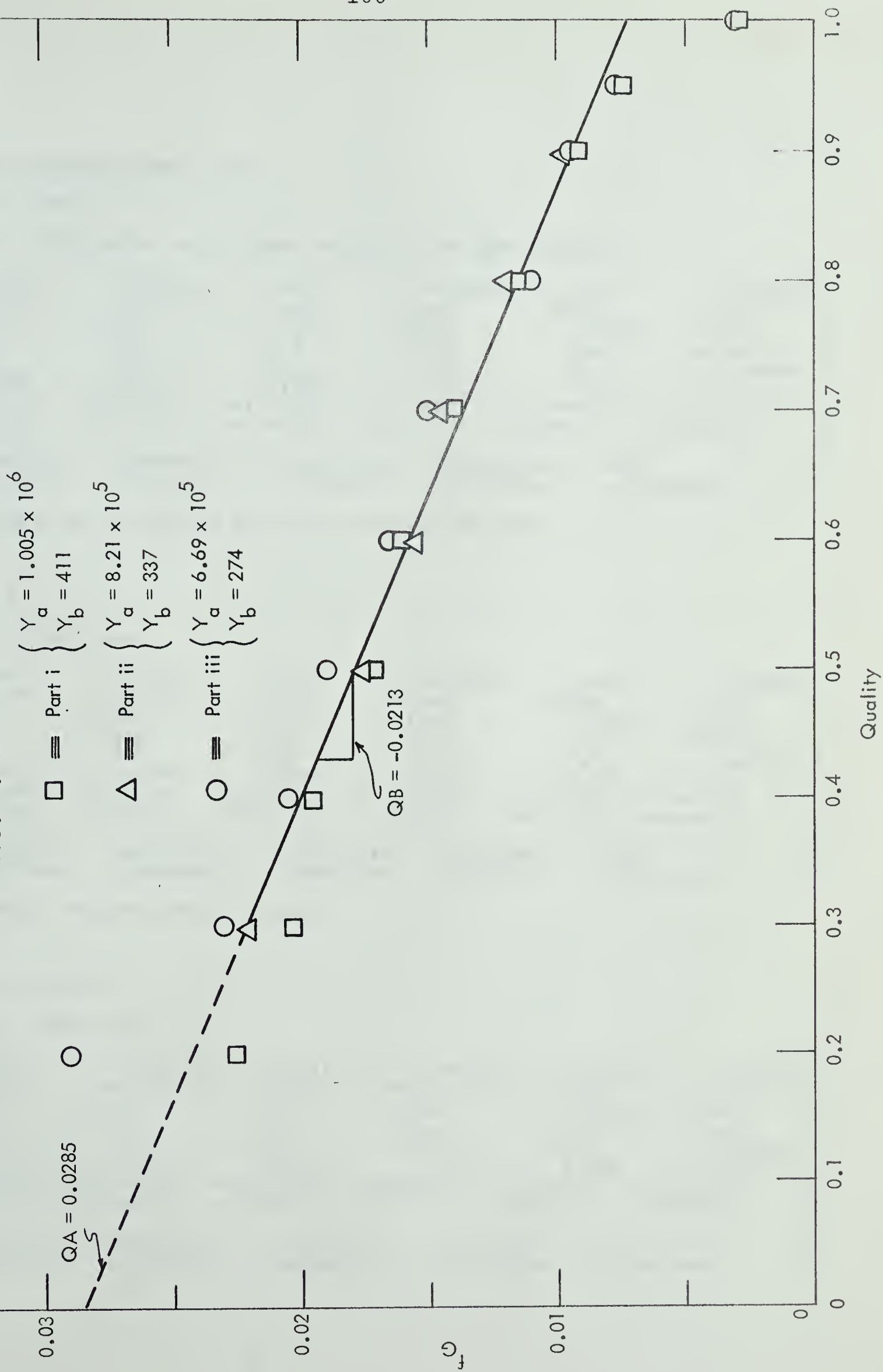


Table II

Y	Part i			Part ii			Part iii		
	$R_{EG} \times 10^{-5}$	γ	f_G	$R_{EG} \times 10^{-5}$	γ	f_G	$R_{EG} \times 10^{-5}$	γ	f_G
0.2	2.01	329	0.025	1.64	270	0.025	1.34	219	0.0290
0.3	3.02	288	0.020	2.46	236	0.022	2.01	192	0.0230
0.4	4.02	247	0.0195	3.28	202	0.020	2.68	164	0.0205
0.5	5.03	206	0.017	4.10	169	0.0175	3.35	137	0.0190
0.6	6.03	164	0.016	4.92	135	0.0155	4.01	110	0.0165
0.7	7.04	123	0.014	5.74	101	0.0145	4.68	82.2	0.0150
0.8	8.04	82.2	0.0115	6.56	67.4	0.012	5.35	54.8	0.0110
0.9	9.05	41.1	0.0091	7.38	33.7	0.0095	6.02	27.4	0.0094
0.95	9.55	20.6	0.0074	7.79	16.9	0.008	6.36	13.7	0.0077
1.0	10.1	0	0.0028	8.20	0	0.0029	6.69	0	0.0030

The friction factors from all three parts are plotted against quality on the same graph.
The scatter of points was such that one straight line could reasonably fit all three parts.

FIG. 17. TWO PHASE FRICTION FACTOR VERSUS QUALITY



III-B COMPUTER INPUT DATA

FIG 2 AND 3

BASIC INPUT DATA (ALSO INPUT RESULTING IN SAMPLE OUTPUT)

0.239	0.000018	0.013	0.000026	0.04	0.000056	
0.0285	-0.0218	1.00	5000.0	927.0	14.7	2
1.0	110.0	0.197	535.0	0.0110	755.0	3
0.8	0.8	0.8	0.8	3.65	6000.0	4
2000.0	0.5	25.0	25.0	25.0	0.00001	5
1.2205	1.4375	2.9945	3.3125	5.025	5.375	6
0.500E-01	0.100E-01	0.100E+00	0.100E-03	0.100E-02	0.100E-04	7
0.100E+04	0.250E+02	0.100E+02	0.0	9.278E-01	4.064E+02	8
5 1 4						9
-0.96816024	-0.83603111	-0.61337143	-0.32425342	-0.0		10
0.08127439	0.18064816	0.26061070	0.31234708	0.33023936		11

CHANGES: QB TO -0.0225 AND WST TO 3000.0 ON CARD 2

FIG 4

BASIC INPUT DATA

0.239	0.000018	0.013	0.000026	0.040	0.000056	
0.0285	-0.0218	0.90	4850.0	862.0	14.7	2
1.4	140.0	0.23	518.0	0.0263	824.1	3
0.8	0.8	0.8	0.8	2.0	1637.0	4
2000.0	0.3	25.0	25.0	25.0	0.00001	5
1.2205	1.4375	1.979	2.250	5.025	5.375	6
0.300E-01	0.100E-01	0.100E+00	0.100E-03	0.100E-02	0.100E-04	7
0.500E+01	0.250E+02	0.100E+01	0.100E+01	1.823E+00	3.500E+03	8
5 1 4						9
-0.96816024	-0.83603111	-0.61337143	-0.32425342	-0.0		10
0.08127439	0.18064816	0.26061070	0.31234708	0.33023936		11

CHANGES: QUAL ON CARD 2 TO 0.8

FIG 5,6 AND 7

BASIC INPUT DATA

0.240	0.000023	0.0133	0.000021	0.0395	0.000050	
0.0285	-0.0213	0.80	10500.0	1020.0	14.7	2
0.4	140.0	0.23	520.0	0.0108	624.2	3
0.8	0.8	0.8	0.8	1.00	3250.0	4
2000.0	0.5	25.0	25.0	25.0	0.00001	5
0.9975	1.1875	2.522	2.750	5.025	5.375	6
0.500E-01	0.100E-01	0.100E+00	0.100E-03	0.100E-02	0.100E-04	7
0.100E+05	0.250E+02	0.100E+02	0.0	3.868E-01	5.342E+02	8
5 1 4						9
-0.96816024	-0.83603111	-0.61337143	-0.32425342	-0.0		10
0.08127439	0.18064816	0.26061070	0.31234708	0.33023936		11

CHANGES: R1 TO 1.1250 AND R2 TO 1.3125 ON CARD 6
 R1 TO 1.2205 AND R2 TO 1.4375 ON CARD 6
 R1 TO 1.3750 AND R2 TO 1.6250 ON CARD 6
 R1 TO 1.4960 AND R2 TO 1.7500 ON CARD 6

FIG 8 AND 9

BASIC INPUT DATA

0.240	0.000023	0.0133	0.000021	0.0395	0.000050	
0.0285	-0.0218	0.80	8000.0	1047.0	14.7	2
0.4	140.0	0.23	520.0	0.0108	574.7	3
0.8	0.8	0.8	0.8	1.00	3250.0	4
2000.0	0.300	25.0	25.0	25.0	0.00001	5
1.22055	1.4375	2.045	2.250	5.025	5.375	6
0.500E-01	0.100E-01	0.100E+00	0.100E-02	0.100E-01	0.100E-04	7
0.100E+04	0.100E+03	0.100E+02	0.0	3.012E-01	5.707E+02	8
5 1 1						9
-0.96816024	-0.83603111	-0.61337143	-0.32425342	-0.0		10
0.08127439	0.18064816	0.26061070	0.31234708	0.33023936		11

CHANGES: WST ON CARD 2 TO 10500, 12000, 14000, 18000

FIG 10,11,12 AND 13

BASIC INPUT DATA

0.240	0.000023	0.0133	0.000021	0.0395	0.000050	
0.0285	-0.0218	0.80	10500.0	1020.0	14.7	2
0.4	140.0	0.23	520.0	0.0108	624.2	3
0.8	0.8	0.8	0.8	1.00	3250.0	4
2000.0	0.5	25.0	25.0	25.0	0.00001	5
0.9975	1.1875	2.045	2.250	5.025	5.375	6
0.500E-01	0.100E-01	0.100E+00	0.100E-03	0.100E-02	0.100E-04	7
0.100E+05	0.250E+02	0.100E+02	0.0	3.868E-01	5.342E+02	8
5 1 4						9
-0.96816024	-0.83603111	-0.61337143	-0.32425342	-0.0		10
0.08127439	0.18064816	0.26061070	0.31234708	0.33023936		11

CHANGES: STEMP1 ON CARD 2 TO 1040.0 , 1060.0 , 1080.0 , 1100.0
 HEVAP ON CARD 3 TO 588.4 , 548.5 , 503.6 , 452.0
 VGI ON CARD 8 TO 3.217E-01 , 2.668E-01 , 2.201E-01 , 1.798E-01
 HRLIQ ON CARD 8 TO 5.609E+02 , 5.890E+02 , 6.187E+02 , 6.506E+02

NOTE: ONE COLUMN REPRESENTS ONE DIFFERENT RUN

THE CHANGES FOR FIG 13 ARE STATED ON THE GRAPH

III-C SAMPLE COMPUTER OUTPUT (FOR FIG 2 AND 3)

WELLBORE AFTER

3.65 DAYS

DEPTH (FT)	STEAM TEMP (F)	CUM HEAT LOSS (BTU/HR)	QUALITY	CASING TEMP (F)	F. LOSS
0.00	467.0000	0.0000E-38	1.0000	279.1233	0.0000
167.06	466.0918	0.8603E 05	0.9774	279.3994	0.0148
266.52	465.5360	0.1369E 06	0.9641	279.5535	0.0236
366.00	464.9684	0.1876E 06	0.9509	279.6992	0.0323
465.54	464.3894	0.2381E 06	0.9378	279.8375	0.0410
565.12	463.7999	0.2883E 06	0.9247	279.9689	0.0496
664.73	463.2004	0.3383E 06	0.9118	280.0936	0.0583
764.37	462.5920	0.3881E 06	0.8989	280.2124	0.0668
864.04	461.9749	0.4377E 06	0.8861	280.3257	0.0754
963.74	461.3501	0.4870E 06	0.8735	280.4342	0.0839
1063.47	460.7178	0.5360E 06	0.8609	280.5383	0.0923
1163.23	460.0790	0.5849E 06	0.8483	280.6384	0.1007
1263.02	459.4342	0.6335E 06	0.8359	280.7350	0.1091
1362.82	458.7841	0.6819E 06	0.8236	280.8287	0.1174
1462.66	458.1292	0.7300E 06	0.8113	280.9198	0.1257
1562.51	457.4701	0.7779E 06	0.7992	281.0090	0.1340
1662.39	456.8075	0.8255E 06	0.7871	281.0967	0.1422
1762.29	456.1419	0.8729E 06	0.7752	281.1831	0.1503
1862.22	455.4738	0.9200E 06	0.7633	281.2687	0.1584
1962.16	454.8039	0.9669E 06	0.7515	281.3543	0.1665
2062.12	454.1327	0.1014E 07	0.7398	281.4396	0.1745
2162.11	453.4608	0.1060E 07	0.7282	281.5258	0.1825
2262.11	452.7886	0.1106E 07	0.7166	281.6128	0.1905
2362.13	452.1169	0.1152E 07	0.7052	281.7012	0.1984
2462.18	451.4458	0.1198E 07	0.6938	281.7913	0.2062
2562.24	450.7762	0.1243E 07	0.6826	281.8834	0.2141
2662.33	450.1084	0.1288E 07	0.6714	281.9779	0.2218
2762.43	449.4431	0.1333E 07	0.6603	282.0753	0.2296
2862.55	448.7806	0.1378E 07	0.6493	282.1758	0.2373
2962.69	448.1215	0.1422E 07	0.6383	282.2799	0.2449
3062.85	447.4663	0.1466E 07	0.6275	282.3877	0.2525
3163.02	446.8155	0.1510E 07	0.6167	282.4998	0.2601
3263.22	446.1693	0.1554E 07	0.6060	282.6164	0.2676
3363.44	445.5285	0.1598E 07	0.5954	282.7377	0.2751
3463.68	444.8935	0.1641E 07	0.5849	282.8643	0.2825
3563.93	444.2646	0.1684E 07	0.5744	282.9961	0.2899
3664.21	443.6424	0.1726E 07	0.5641	283.1336	0.2973
3764.51	443.0272	0.1769E 07	0.5538	283.2773	0.3046
3864.82	442.4195	0.1811E 07	0.5435	283.4270	0.3119
3965.17	441.8196	0.1853E 07	0.5334	283.5834	0.3191
4065.53	441.2280	0.1895E 07	0.5233	283.7466	0.3263
4165.91	440.6450	0.1937E 07	0.5133	283.9168	0.3335
4266.32	440.0712	0.1978E 07	0.5033	284.0942	0.3406
4366.74	439.5067	0.2019E 07	0.4934	284.2791	0.3477
4467.20	438.9519	0.2060E 07	0.4836	284.4717	0.3547
4567.66	438.4074	0.2101E 07	0.4738	284.6722	0.3618
4668.17	437.8731	0.2141E 07	0.4642	284.8809	0.3687

WELLBORE AFTER

3.65 DAYS

DEPTH (FT)	STEAM TEMP (F)	CUM HEAT LOSS (BTU/HR)	QUALITY	CASING TEMP (F)	F. LOSS
4768.70	437.3496	0.2181E 07	0.4545	285.0978	0.3757
4869.24	436.8372	0.2221E 07	0.4449	285.3232	0.3825
4969.82	436.3360	0.2261E 07	0.4354	285.5571	0.3894
5070.42	435.8465	0.2301E 07	0.4260	285.7048	0.3962
5171.06	435.3688	0.2340E 07	0.4166	285.9572	0.4030
5271.71	434.9032	0.2380E 07	0.4072	286.2184	0.4098
5372.40	434.4499	0.2419E 07	0.3979	286.4888	0.4165
5473.12	434.0090	0.2457E 07	0.3886	286.7686	0.4232
5573.87	433.5809	0.2496E 07	0.3794	287.0578	0.4298
5674.65	433.1656	0.2534E 07	0.3703	287.3562	0.4364
5775.48	432.7633	0.2573E 07	0.3612	287.6644	0.4430
5876.32	432.3741	0.2611E 07	0.3521	287.9818	0.4496
5977.22	431.9981	0.2648E 07	0.3431	288.3092	0.4561
5999.99	431.9151	0.2657E 07	0.3411	288.3843	0.4575

WELLBORE AFTER

36.50 DAYS

0.00	467.0000	0.0000E-38	1.0000	318.0724	0.0000
167.25	466.0928	0.7083E 05	0.9814	318.0571	0.0122
266.74	465.5395	0.1127E 06	0.9705	318.0373	0.0194
366.26	464.9753	0.1544E 06	0.9597	318.0090	0.0266
465.81	464.4002	0.1959E 06	0.9489	317.9723	0.0337
565.38	463.8149	0.2372E 06	0.9382	317.9276	0.0409
664.98	463.2197	0.2784E 06	0.9276	317.8757	0.0479
764.60	462.6151	0.3193E 06	0.9170	317.8164	0.0550
864.24	462.0014	0.3601E 06	0.9065	317.7506	0.0620
963.90	461.3790	0.4006E 06	0.8961	317.6782	0.0690
1063.59	460.7483	0.4410E 06	0.8858	317.5999	0.0759
1163.29	460.1097	0.4812E 06	0.8755	317.5159	0.0829
1263.00	459.4635	0.5211E 06	0.8653	317.4265	0.0897
1362.73	458.8101	0.5609E 06	0.8552	317.3320	0.0966
1462.48	458.1499	0.6004E 06	0.8452	317.2330	0.1034
1562.25	457.4831	0.6398E 06	0.8352	317.1295	0.1102
1662.03	456.8102	0.6789E 06	0.8253	317.0220	0.1169
1761.82	456.1314	0.7179E 06	0.8155	316.9107	0.1236
1861.63	455.4471	0.7566E 06	0.8058	316.7958	0.1303
1961.44	454.7576	0.7952E 06	0.7961	316.6778	0.1369
2061.28	454.0632	0.8335E 06	0.7866	316.5568	0.1435
2161.12	453.3643	0.8716E 06	0.7771	316.4332	0.1501
2260.98	452.6611	0.9095E 06	0.7676	316.3073	0.1566
2360.84	451.9539	0.9472E 06	0.7583	316.1793	0.1631
2460.72	451.2432	0.9847E 06	0.7490	316.0495	0.1696
2560.61	450.5291	0.1022E 07	0.7398	315.9182	0.1760
2660.51	449.8119	0.1059E 07	0.7307	315.7855	0.1824
2760.42	449.0920	0.1096E 07	0.7216	315.6516	0.1887
2860.34	448.3696	0.1133E 07	0.7127	315.5173	0.1950
2960.27	447.6450	0.1169E 07	0.7038	315.3821	0.2013
3060.21	446.9185	0.1205E 07	0.6950	315.2466	0.2075

WELLBORE AFTER

36.50 DAYS

DEPTH (FT)	STEAM TEMP (F)	CUM HEAT LOSS (BTU/HR)	QUALITY	CASING TEMP (F)	F. LOSS
3160.16	446.1903	0.1241E 07	0.6862	315.1113	0.2137
3260.12	445.4608	0.1277E 07	0.6775	314.9759	0.2199
3360.09	444.7303	0.1313E 07	0.6689	314.8409	0.2260
3460.08	443.9989	0.1348E 07	0.6604	314.7065	0.2321
3560.05	443.2671	0.1383E 07	0.6520	314.5731	0.2382
3660.05	442.5348	0.1418E 07	0.6436	314.4406	0.2442
3760.06	441.8026	0.1453E 07	0.6353	314.2577	0.2502
3860.07	441.0707	0.1487E 07	0.6270	314.1283	0.2561
3960.09	440.3393	0.1522E 07	0.6189	314.0005	0.2620
4060.13	439.6087	0.1556E 07	0.6108	313.8746	0.2679
4160.17	438.8792	0.1590E 07	0.6027	313.7509	0.2737
4260.22	438.1509	0.1623E 07	0.5948	313.6294	0.2795
4360.28	437.4242	0.1657E 07	0.5869	313.5105	0.2853
4460.34	436.6993	0.1690E 07	0.5791	313.3943	0.2910
4560.42	435.9764	0.1723E 07	0.5713	313.2809	0.2967
4660.50	435.2559	0.1756E 07	0.5636	313.1707	0.3024
4760.60	434.5378	0.1789E 07	0.5560	313.0636	0.3080
4860.70	433.8226	0.1821E 07	0.5485	312.9601	0.3136
4960.81	433.1105	0.1853E 07	0.5410	312.8602	0.3191
5060.93	432.4016	0.1885E 07	0.5336	312.7642	0.3247
5161.06	431.6962	0.1917E 07	0.5262	312.6721	0.3301
5261.20	430.9946	0.1949E 07	0.5189	312.5841	0.3356
5361.35	430.2969	0.1980E 07	0.5117	312.5005	0.3410
5461.51	429.6036	0.2011E 07	0.5045	312.4215	0.3464
5561.68	428.9146	0.2042E 07	0.4974	312.3471	0.3517
5661.85	428.2305	0.2073E 07	0.4904	312.2776	0.3570
5762.03	427.5511	0.2104E 07	0.4834	312.2130	0.3623
5862.23	426.8769	0.2134E 07	0.4764	312.1536	0.3675
5962.44	426.2081	0.2164E 07	0.4696	312.0995	0.3727
6000.01	425.9589	0.2175E 07	0.4670	312.0807	0.3746

WELLBORE AFTER

365.00 DAYS

0.00	467.0000	0.0000E-38	1.0000	344.3480	0.0000
167.38	466.0936	0.5985E 05	0.9843	344.1456	0.0103
266.91	465.5422	0.9521E 05	0.9751	344.0148	0.0164
366.46	464.9805	0.1304E 06	0.9660	343.8755	0.0225
466.02	464.4086	0.1655E 06	0.9569	343.7279	0.0285
565.60	463.8268	0.2004E 06	0.9479	343.5721	0.0345
665.20	463.2354	0.2351E 06	0.9390	343.4085	0.0405
764.82	462.6345	0.2697E 06	0.9301	343.2371	0.0464
864.45	462.0245	0.3041E 06	0.9212	343.0583	0.0524
964.09	461.4055	0.3383E 06	0.9125	342.8724	0.0583
1063.75	460.7777	0.3724E 06	0.9038	342.6797	0.0641
1163.42	460.1414	0.4063E 06	0.8951	342.4802	0.0700
1263.10	459.4968	0.4401E 06	0.8866	342.2741	0.0758
1362.80	458.8440	0.4736E 06	0.8780	342.0618	0.0816
1462.50	458.1833	0.5070E 06	0.8696	341.8433	0.0873

WELLBORE AFTER 365.00 DAYS

DEPTH (FT)	STEAM TEMP (F)	CUM HEAT LOSS (BTU/HR)	QUALITY	CASING TEMP (F)	F. LOSS
1562.22	457.5148	0.5403E 06	0.8612	341.6189	0.0930
1661.94	456.8387	0.5733E 06	0.8529	341.3889	0.0987
1761.67	456.1552	0.6062E 06	0.8446	341.1533	0.1044
1861.42	455.4645	0.6389E 06	0.8364	340.9123	0.1100
1961.17	454.7667	0.6714E 06	0.8283	340.6660	0.1156
2060.93	454.0620	0.7038E 06	0.8203	340.4149	0.1212
2160.69	453.3505	0.7360E 06	0.8123	340.1588	0.1267
2260.47	452.6322	0.7680E 06	0.8043	339.8979	0.1322
2360.25	451.9076	0.7998E 06	0.7965	339.6326	0.1377
2460.04	451.1766	0.8314E 06	0.7887	339.3629	0.1432
2559.83	450.4395	0.8629E 06	0.7809	339.0889	0.1486
2659.63	449.6962	0.8942E 06	0.7733	338.8108	0.1540
2759.43	448.9470	0.9253E 06	0.7657	338.5286	0.1593
2859.24	448.1920	0.9562E 06	0.7582	338.2424	0.1647
2959.06	447.4313	0.9869E 06	0.7507	337.9528	0.1700
3058.88	446.6650	0.1017E 07	0.7433	337.6594	0.1752
3158.70	445.8932	0.1048E 07	0.7360	337.3626	0.1804
3258.53	445.1160	0.1078E 07	0.7287	337.0622	0.1856
3358.36	444.3335	0.1108E 07	0.7215	336.7309	0.1908
3458.20	443.5458	0.1138E 07	0.7144	336.4243	0.1959
3558.04	442.7530	0.1168E 07	0.7073	336.1147	0.2011
3657.88	441.9552	0.1197E 07	0.7003	335.8021	0.2061
3757.73	441.1524	0.1226E 07	0.6934	335.4865	0.2112
3857.58	440.3448	0.1255E 07	0.6865	335.1682	0.2162
3957.44	439.5324	0.1284E 07	0.6797	334.8472	0.2211
4057.29	438.7153	0.1313E 07	0.6730	334.5234	0.2261
4157.15	437.8935	0.1341E 07	0.6663	334.1973	0.2310
4257.02	437.0671	0.1370E 07	0.6598	333.8685	0.2359
4356.88	436.2362	0.1398E 07	0.6532	333.5372	0.2407
4456.75	435.4008	0.1426E 07	0.6468	333.2038	0.2455
4556.61	434.5610	0.1454E 07	0.6404	332.8679	0.2503
4656.48	433.7167	0.1481E 07	0.6340	332.5298	0.2551
4756.36	432.8681	0.1508E 07	0.6278	332.1895	0.2598
4856.23	432.0152	0.1536E 07	0.6216	331.8472	0.2644
4956.10	431.1580	0.1563E 07	0.6154	331.5027	0.2691
5055.98	430.2964	0.1589E 07	0.6094	331.1560	0.2737
5155.86	429.4307	0.1616E 07	0.6034	330.8075	0.2783
5255.73	428.5606	0.1642E 07	0.5974	330.4567	0.2828
5355.61	427.6864	0.1669E 07	0.5916	330.1042	0.2873
5455.49	426.8079	0.1695E 07	0.5858	329.7497	0.2918
5555.37	425.9251	0.1720E 07	0.5800	329.3931	0.2963
5655.25	425.0381	0.1746E 07	0.5743	329.0348	0.3007
5755.13	424.1468	0.1772E 07	0.5687	328.6743	0.3051
5855.00	423.2512	0.1797E 07	0.5632	328.2875	0.3094
5954.88	422.3513	0.1822E 07	0.5577	327.9237	0.3137
6000.02	421.9435	0.1833E 07	0.5552	327.7587	0.3157

WELLBORE AFTER 3650.00 DAYS

DEPTH (FT)	STEAM TEMP (F)	CUM HEAT LOSS (BTU/HR)	QUALITY	CASING TEMP (F)	F. LOSS
0.00	467.0000	0.0000E-38	1.0000	362.9467	0.0000
167.49	466.0941	0.5168E 05	0.9865	362.6183	0.0089
267.04	465.5442	0.8222E 05	0.9786	362.4132	0.0142
366.61	464.9845	0.1126E 06	0.9707	362.1995	0.0194
466.19	464.4151	0.1429E 06	0.9629	361.9776	0.0246
565.78	463.8362	0.1730E 06	0.9551	361.7477	0.0298
665.39	463.2480	0.2030E 06	0.9474	361.5096	0.0350
765.01	462.6505	0.2328E 06	0.9398	361.2639	0.0401
864.64	462.0438	0.2625E 06	0.9322	361.0103	0.0452
964.27	461.4282	0.2921E 06	0.9246	360.7492	0.0503
1063.92	460.8036	0.3215E 06	0.9171	360.4809	0.0554
1163.58	460.1703	0.3508E 06	0.9097	360.2050	0.0604
1263.24	459.5282	0.3799E 06	0.9023	359.9221	0.0654
1362.92	458.8776	0.4089E 06	0.8950	359.6321	0.0704
1462.60	458.2184	0.4377E 06	0.8877	359.3350	0.0754
1562.28	457.5509	0.4664E 06	0.8805	359.0312	0.0803
1661.98	456.8749	0.4950E 06	0.8733	358.7206	0.0852
1761.68	456.1907	0.5234E 06	0.8662	358.4034	0.0901
1861.38	455.4983	0.5516E 06	0.8591	358.0795	0.0950
1961.09	454.7977	0.5797E 06	0.8522	357.7492	0.0998
2060.81	454.0890	0.6076E 06	0.8452	357.4124	0.1046
2160.53	453.3722	0.6354E 06	0.8383	357.0694	0.1094
2260.25	452.6475	0.6631E 06	0.8315	356.7200	0.1142
2359.98	451.9147	0.6905E 06	0.8248	356.3644	0.1189
2459.71	451.1740	0.7179E 06	0.8180	356.0026	0.1236
2559.45	450.4254	0.7450E 06	0.8114	355.6348	0.1283
2659.19	449.6688	0.7720E 06	0.8048	355.2609	0.1329
2758.92	448.9042	0.7989E 06	0.7983	354.8808	0.1376
2858.68	448.1315	0.8256E 06	0.7918	354.4946	0.1422
2958.43	447.3512	0.8521E 06	0.7854	354.1026	0.1467
3058.18	446.5629	0.8785E 06	0.7791	353.7047	0.1513
3157.93	445.7667	0.9047E 06	0.7728	353.2851	0.1558
3257.68	444.9625	0.9308E 06	0.7665	352.8751	0.1603
3357.43	444.1502	0.9567E 06	0.7604	352.4594	0.1647
3457.19	443.3299	0.9824E 06	0.7542	352.0376	0.1692
3556.94	442.5015	0.1008E 07	0.7482	351.6100	0.1736
3656.70	441.6650	0.1033E 07	0.7422	351.1762	0.1780
3756.45	440.8202	0.1059E 07	0.7363	350.7365	0.1823
3856.21	439.9671	0.1084E 07	0.7304	350.2907	0.1866
3955.96	439.1056	0.1109E 07	0.7246	349.8388	0.1909
4055.72	438.2356	0.1133E 07	0.7188	349.3807	0.1952
4155.47	437.3570	0.1158E 07	0.7132	348.9164	0.1994
4255.22	436.4697	0.1182E 07	0.7075	348.4458	0.2036
4354.97	435.5736	0.1207E 07	0.7020	347.9690	0.2078
4454.72	434.6685	0.1231E 07	0.6965	347.4855	0.2119
4554.47	433.7542	0.1255E 07	0.6910	346.9956	0.2160
4654.22	432.8307	0.1278E 07	0.6857	346.4990	0.2201
4753.96	431.8977	0.1302E 07	0.6803	345.9955	0.2242
4853.70	430.9550	0.1325E 07	0.6751	345.4853	0.2282

WELLBORE AFTER 3650.00 DAYS

DEPTH (FT)	STEAM TEMP (F)	CUM HEAT LOSS (BTU/HR)	QUALITY	CASING TEMP (F)	F. LOSS
4953.44	430.0024	0.1348E 07	0.6699	344.9680	0.2322
5053.17	429.0398	0.1371E 07	0.6648	344.4435	0.2362
5152.90	428.0669	0.1394E 07	0.6597	343.9116	0.2401
5252.63	427.0833	0.1417E 07	0.6547	343.3722	0.2440
5352.35	426.0889	0.1440E 07	0.6498	342.8250	0.2479
5452.07	425.0834	0.1462E 07	0.6449	342.2556	0.2517
5551.78	424.0664	0.1484E 07	0.6401	341.6925	0.2556
5651.49	423.0375	0.1506E 07	0.6354	341.1210	0.2593
5751.19	421.9966	0.1528E 07	0.6307	340.5407	0.2631
5850.89	420.9430	0.1549E 07	0.6261	339.9515	0.2668
5950.58	419.8764	0.1571E 07	0.6216	339.3529	0.2705
6000.00	419.3427	0.1581E 07	0.6194	339.0526	0.2723



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